Random thoughts on coverings

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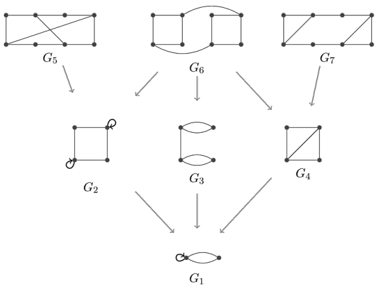
Mazurkiewicz's algorithm (here: asynchronous message passing with port #)



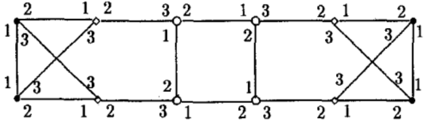
(port numbers preserving symmetries (omitted) – disconnected graphs omitted – bidirectional arcs as edges)

Can we recognize the underlying graph G in the following cases? (knowing n)

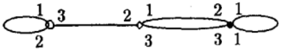
1. G is G_7 $(n = 8)$
2. $n = 8$ and $\mathcal{M}(G)$ computes G_4
3. $n = 8$ and $\mathcal{M}(G)$ computes G_3
4. G is G_1 $(n = 4)$

Remark: Computing the views always gives us G_1 (does not exploit asynchrony!)

Yamashita and Kameda's quotient graphs (versus coverings?)



(a)



(b)

LEMMA 11. *A graph G has a cycle if, for any local edge labeling* $\mathbf{f}$, *$G/\mathbf{f}$ has one of the following:*

1) *a self-loop $(T_i, T_i: p, q)$, $(p \neq q)$,*
2) *two self-loops, $(T_i, T_i: p, p)$ and $(T_j, T_j: q, q)$, where $p = q$ is not excluded if $i \neq j$, or*
3) *a cycle of length ≥ 2.*

LEMMA 9. *Given a graph G and a local edge labeling* $\mathbf{f}$ *for G, let $G/\mathbf{f} = (T_\mathbf{f}, E_\mathbf{f})$. Consider any edge $e = (T_1, T_2: p, q) \in E_\mathbf{f}$, and let $V_1 = \{u \mid T(u) \equiv T_1\}$ and $V_2 = \{u \mid T(u) \equiv T_2\}$.*

1) *If $T_1 = T_2$, i.e., e is a self-loop, and $p = q$, then $F_1 = \left(V_1, \tau_\mathbf{f}^{-1}(e)\right)$ is a 1-factor of G_{V_1}.*
2) *If $T_1 = T_2$, i.e., e is a self-loop, and $p \neq q$, then $F_2 = \left(V_1, \tau_\mathbf{f}^{-1}(e)\right)$ is a 2-factor of G_{V_1}.*
3) *If $T_1 \neq T_2$, then $F_3 = \left(V_1 \mathbin{//} V_2, \tau_\mathbf{f}^{-1}(e)\right)$ is a 1-regular bipartite graph.*

Given a quotient graph $Q = G'/\mathbf{f}' = (W, A)$, where $W = \{w_1, \ldots, w_g\}$, for each $w_i \in W$, define three parameters, ℓ_i, m_i and $\hat{m}_i$, as follows:

- ℓ_i: the number of self-loops of the form $(w_i, w_i: p, p)$ for some w_i and p,
- m_i: the number of self-loops of the form $(w_i, w_i: p, q)$ for some w_i and p, q $(p \neq q)$.
- $\hat{m}_i = \max_{j(\neq 1)}\{m_{ij}\}$, where m_{ij} is the number of parallel edges connecting w_i and w_j.

Define further

$$m = \max_{w_i \in W}\{1 + l_i + 2m_i, \hat{m}_i\},$$

and

$$M = \begin{cases} m & \text{if } m \text{ is even or } l_i = 0 \text{ for all } w_i \in W, \\ m+1 & \text{otherwise}. \end{cases}$$

A conjecture on neighbor graphs?

Neighbor graphs: $G_1 = G_2 \pm e$.

Conjecture: If G_1 is non-minimal, then G_2 is minimal or quasi-minimal

(quasi-minimal = only divisible by 2)

By contradiction, suppose $G_1 \xrightarrow{k_1} H_1$ and $G_2 \xrightarrow{k_2} H_2$, with $k \geq 3$.

$\Longrightarrow$ H_1 and H_2 have different densities

$\Longrightarrow$ some “pattern” in H_2 does not exist in H_1 (nor in G_1) // unclear

$\Longrightarrow$ ≥ 3 disjoint manifestations of this pattern exist in G_2 // unclear

$\Longrightarrow$ Changing an edge can only affect two of them

$\Longrightarrow$ G_1 and G_2 are not neighbors.

Complexity of deciding minimality of a given graph (centralized problem)

Known to be NP-complete.

The above conjecture implies:

If G_1 is non-minimal, then testing minimality of G_2 reduces to testing its divisibility by 2

$\rightarrow$ How hard is this?

Thanks!