Characterizing Topological Assumptions of Distributed Algorithms in Dynamic Networks

A.Casteigts, S.Chaumette, A.Ferreira

June 1, 2009

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Level of abstraction

- Local interactions only.
- Abstraction of the communication model.

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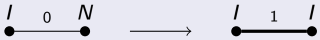

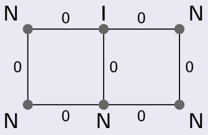

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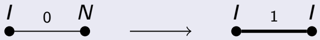

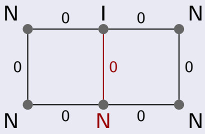

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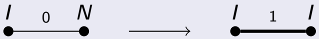
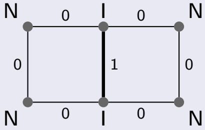
- relabelling rule: 



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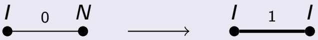

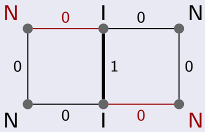

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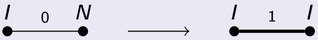
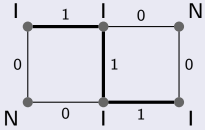
- relabelling rule: 



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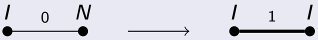

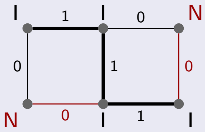

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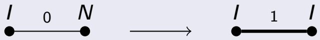
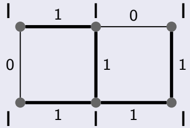
- relabelling rule: 



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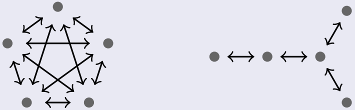

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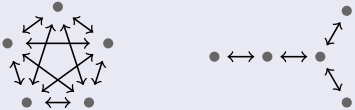

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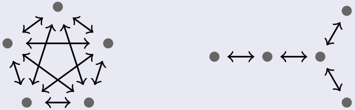

- Characterizing in what context a given thing can be done
  - what properties a given algorithm requires on the graph dynamics? (*i.e.*, on the *topological* evolution)

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Propagating an information

- initial states: I for the initial emitter, N for all the other nodes
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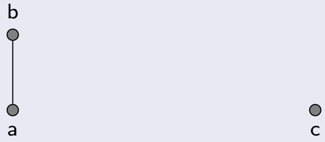

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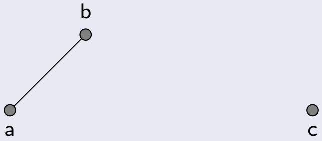

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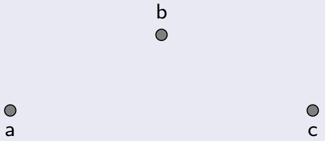

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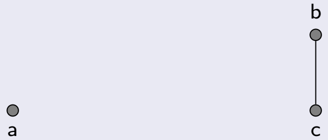

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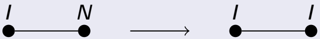

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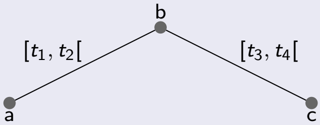
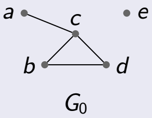


Formalism to represent dynamic topology

Evolving graphs [Ferreira 2004]

period $t_0 \longrightarrow t_1$



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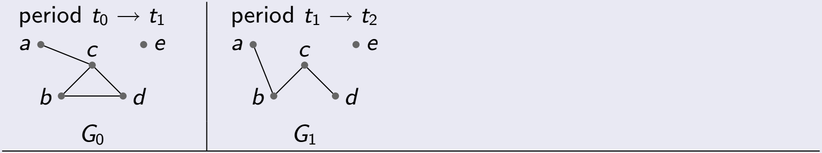

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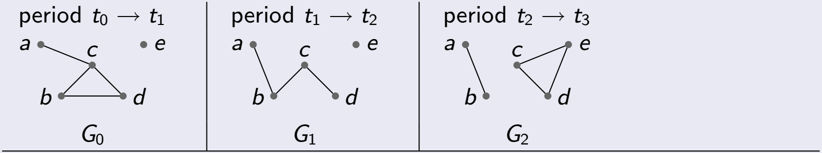

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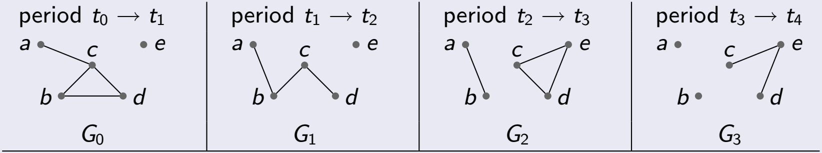

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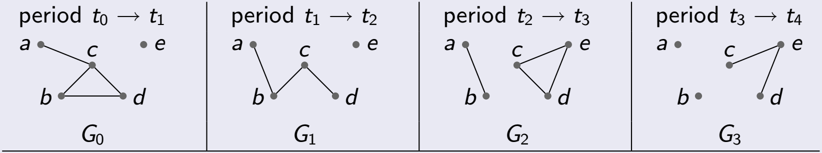

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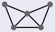
$G = \bigcup_{G_i \in \mathcal{S}_G} =$ 

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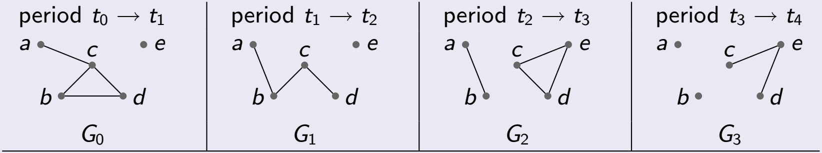

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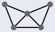

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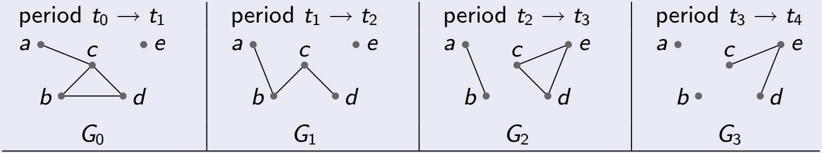

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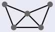
$G = \bigcup_{G_i \in \mathcal{S}_G} =$ 

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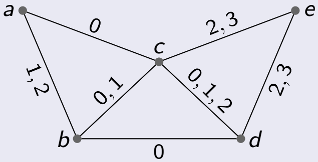

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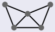
$G = \bigcup_{G_i \in \mathcal{S}_G} =$ 

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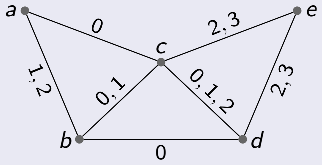

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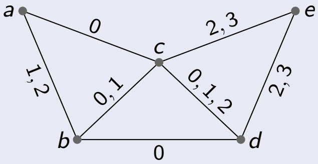

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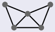
$$G = \bigcup_{G_i \in \mathcal{S}_G} =$$ 

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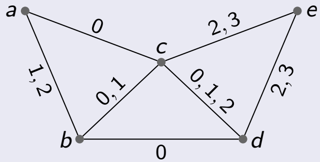

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e.g. Journey (path over time).

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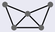

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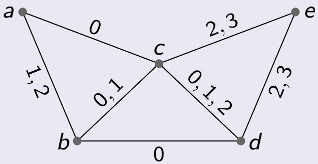

$\mathcal{J}_{a,e} = \{(a, b, 1), (b, c, 1), (c, d, 1), (d, e, 2)\}$
is a journey from a to e.

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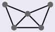

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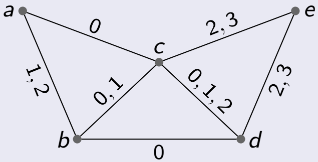


$\mathcal{J}_{a,e} = \{(a,b,1),(b,c,1),(c,d,1),(d,e,2)\}$ is a journey from a to e.

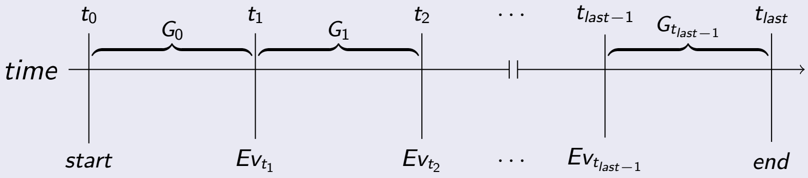
$\mathcal{J}_{a,e} = \{(a,c,0),(c,e,2)\}$ is a strict journey from a to e.

→ possibility to express topological properties, and to define related concepts.

e.g. Journey (path over time).

Combination

Relabellings over Evolving Graphs



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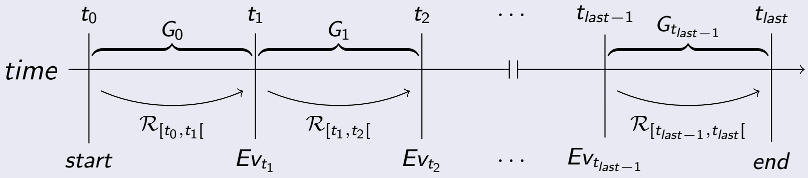

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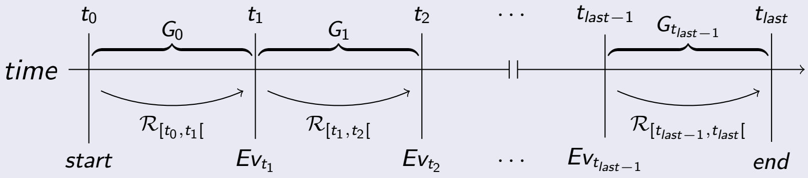
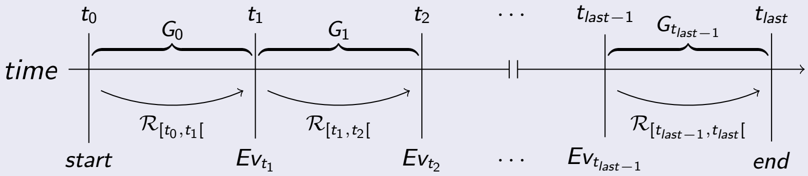


An execution is an alternated sequence of relabellings and topological events:

$$X = \mathcal{R}_{\mathcal{A}_{[t_{last-1},t_{last}[}} \circ Event_{t_{last-1}} \circ .. \circ Event_{t_i} \circ \mathcal{R}_{\mathcal{A}_{[t_{i-1},t_i[}} \circ .. \circ Event_{t_1} \circ \mathcal{R}_{\mathcal{A}_{[t_0,t_1[}}(\mathbf{G}_0)$$

Combination

Relabellings over Evolving Graphs



An execution is an alternated sequence of relabellings and topological events:
$X = \mathcal{R}_{\mathcal{A}_{[t_{last-1},t_{last}[}} \circ Event_{t_{last-1}} \circ .. \circ Event_{t_i} \circ \mathcal{R}_{\mathcal{A}_{[t_{i-1},t_i[}} \circ .. \circ Event_{t_1} \circ \mathcal{R}_{\mathcal{A}_{[t_0,t_1[}}(\mathbf{G}_0)$

We note $\mathcal{X}_{\mathcal{A}/\mathcal{G}}$ the set of all possible execution sequences of an algorithm $\mathcal{A}$ over an evolving graph $\mathcal{G}$

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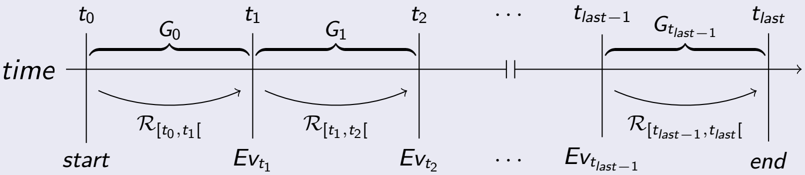

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Topology-related necessary condition: $\neg\mathcal{C}_{\mathcal{N}}(\mathcal{G}) \implies \nexists X \in \mathcal{X}_{\mathcal{A}/\mathcal{G}} \mid success$.
Topology-related sufficient condition: $\mathcal{C}_{\mathcal{S}}(\mathcal{G}) \implies \forall X$ in $\mathcal{X}_{\mathcal{A}/\mathcal{G}}, success$.

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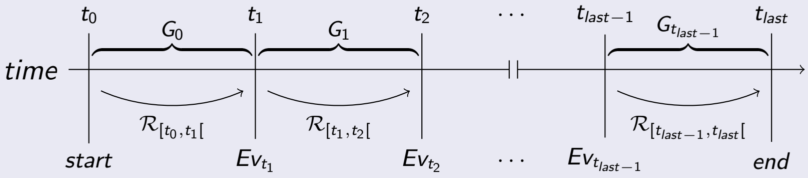

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$\rightarrow$ possibility to prove formally that a given property is necessary, or sufficient.

Classes of evolving graphs

Propagation algorithm conditions

In order to inform all the nodes, it is:

- *necessary* that a journey exists from the emitter to every other node ($\mathcal{C}_{\mathcal{N}}$).

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$\longrightarrow$ $\mathcal{F}_4$: same as $\mathcal{F}_2$ but with strict journeys ($*$-$\mathcal{J}_{strict}$-$*$).

Classes of evolving graphs (2)

Enumeration algorithm with a pre-selected counter

- initial states: $(C, 1)$ for the counter, N for all other vertices.
- relabelling rule: $\overset{C,i}{\bullet}\!\!-\!\!-\!\!-\!\!\overset{N}{\bullet} \longrightarrow \overset{C,i+1}{\bullet}\!\!-\!\!-\!\!-\!\!\overset{F}{\bullet}$
- (N means `non-counted`, F means `counted`)

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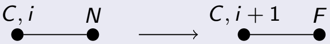
- relabelling rule: 

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- $\mathcal{C}_{\mathcal{N}} = \mathcal{C}_{\mathcal{S}}$: the counter will share an edge with every other node (at possibly various times and durations).

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  - $\longrightarrow$ $\mathcal{F}_6$: graphs where the condition is verified for all nodes (success guaranteed whatever the counter. Also, not being in this class means that at least one node would fail as counter)

Classes of evolving graphs (3)

Decentralized counting algorithm

- initial states: $(C, 1)$ for all vertices.
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$\mathcal{C}_\mathcal{N}$ and resulting class

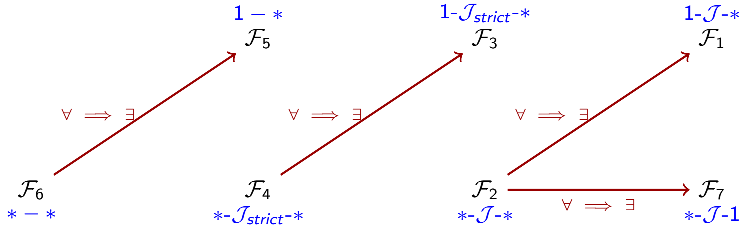
- $\mathcal{C}_\mathcal{N}$: at least one node can be reached by all the others by a journey.

  $\longrightarrow$ $\mathcal{F}_7$: idem.

Classification of dynamic networks

	$1 - *$	$1\text{-}\mathcal{J}_{strict}\text{-}*$	$1\text{-}\mathcal{J}\text{-}*$
	$\mathcal{F}_5$	$\mathcal{F}_3$	$\mathcal{F}_1$
$\mathcal{F}_6$	$\mathcal{F}_4$	$\mathcal{F}_2$	$\mathcal{F}_7$
$* - *$	$*\text{-}\mathcal{J}_{strict}\text{-}*$	$*\text{-}\mathcal{J}\text{-}*$	$*\text{-}\mathcal{J}\text{-}1$

Classification of dynamic networks



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$$
\begin{tikzcd}
 & \overset{1-*}{\mathcal{F}_5} \arrow[r, "- \implies \mathcal{J}_{strict}"] & \overset{1\text{-}\mathcal{J}_{strict}\text{-}*}{\mathcal{F}_3} & \overset{1\text{-}\mathcal{J}\text{-}*}{\mathcal{F}_1} \\
\underset{*-*}{\mathcal{F}_6} \arrow[ur] \arrow[r, "- \implies \mathcal{J}_{strict}"'] & \underset{*\text{-}\mathcal{J}_{strict}\text{-}*}{\mathcal{F}_4} \arrow[ur] & \underset{*\text{-}\mathcal{J}\text{-}*}{\mathcal{F}_2} \arrow[ur] \arrow[r] & \underset{*\text{-}\mathcal{J}\text{-}1}{\mathcal{F}_7}
\end{tikzcd}
$$

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\mathcal{F}_6 \arrow[ur] \arrow[r] & \mathcal{F}_4 \arrow[ur] \arrow[r, "\mathcal{J}_{strict} \implies \mathcal{J}"'] & \mathcal{F}_2 \arrow[ur] \arrow[r] & \mathcal{F}_7 \\
- & *\text{-}\mathcal{J}_{strict}\text{-}* & *\text{-}\mathcal{J}\text{-}* & *\text{-}\mathcal{J}\text{-}1
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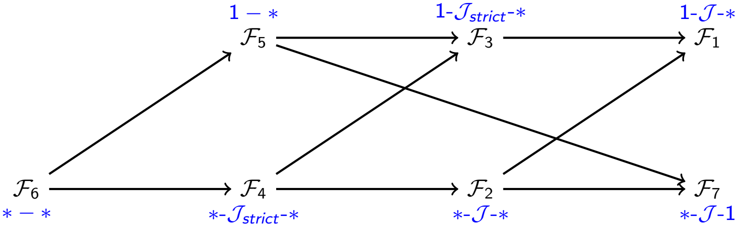

Algorithms Comparison

$$\begin{tikzcd}
 & \mathcal{F}_5 \arrow[r] \arrow[drr] & \mathcal{F}_3 \arrow[r] & \mathcal{F}_1 \\
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C_{\mathcal{N}}(counting_{v1}) \arrow[u] & & C_{\mathcal{N}}(counting_{v2}) \arrow[u] &
\end{tikzcd}$$

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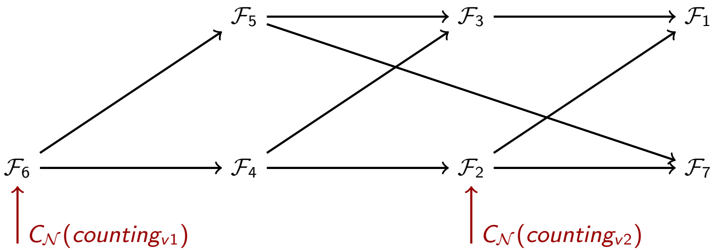

- It exists topologies where $counting_{v1}$ necessarily fails, while $counting_{v2}$ might have some chances of success.

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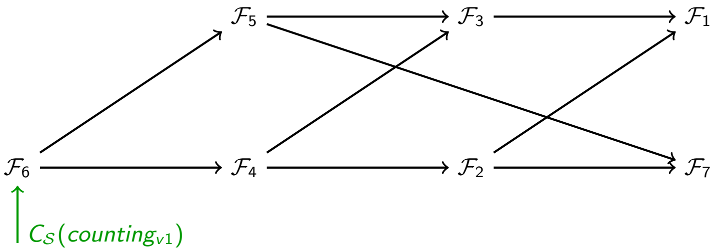

- It exists topologies where *counting*$_{v1}$ necessarily fails, while *counting*$_{v2}$ might have some chances of success.
- But if we know that the condition of *counting*$_{v1}$ is matched, then better using this one.

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- But if we know that the condition of *counting*$_{v1}$ is matched, then better using this one.
- The choice depends on the expected properties of the target context.
- It would be interesting to now what properties the target context is likely to match.

Automated Verification

Mobility Model $\longrightarrow$ Generation $\longrightarrow$ Network Traces

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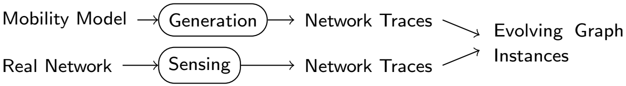
Mobility Model $\longrightarrow$ $\boxed{\text{Generation}}$ $\longrightarrow$ Network Traces

Real Network $\longrightarrow$ $\boxed{\text{Sensing}}$ $\longrightarrow$ Network Traces

Automated Verification



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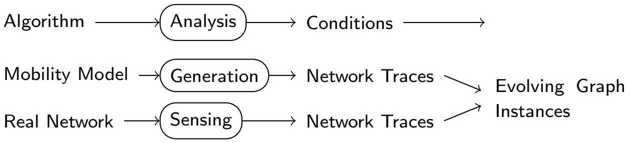


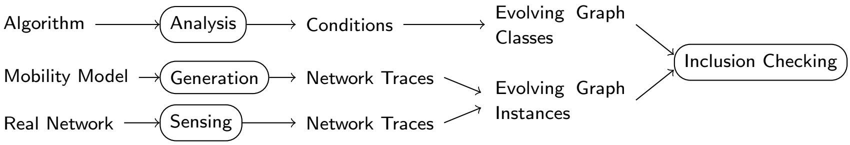
Automated Verification

Algorithm → Analysis → Conditions → Evolving Graph Classes

Mobility Model → Generation → Network Traces → Evolving Graph Instances

Real Network → Sensing → Network Traces → Evolving Graph Instances

Automated Verification



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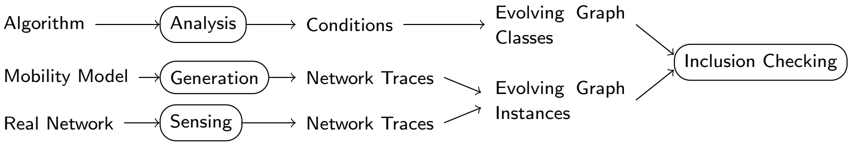

$\mathcal{C}_{\mathcal{N}}$ (or $\mathcal{C}_{\mathcal{S}}$) is matched in a all/none/some cases? $\Longrightarrow$ decision.

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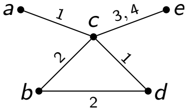

$\longrightarrow$

$\mathcal{G} = (G, \mathcal{S}_G, \mathcal{S}_{\mathbb{T}})$

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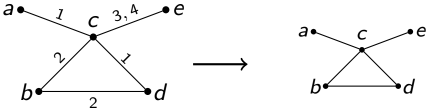

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G

(Underlying graph)

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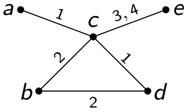

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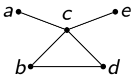

G

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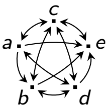
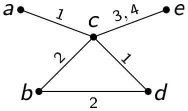


H

(Transitive closure of journeys)

Automated Verification



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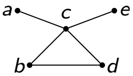

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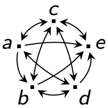

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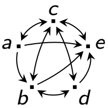
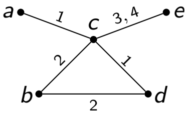


H_{strict}
(Transitive closure of strict journeys)

Automated Verification



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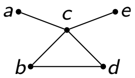

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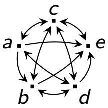

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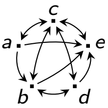

H_{strict}

(Transitive closure of strict journeys)

$\mathcal{G} \in \mathcal{F}_1$	$(1\text{-}\mathcal{J}\text{-}*)$	$\Longleftrightarrow$	H contains an out-dominating set of size 1.
$\mathcal{G} \in \mathcal{F}_2$	$(*\text{-}\mathcal{J}\text{-}*)$	$\Longleftrightarrow$	H is a complete graph.
$\mathcal{G} \in \mathcal{F}_3$	$(1\text{-}\mathcal{J}_{strict}\text{-}*)$	$\Longleftrightarrow$	H_{strict} contains an out-dominating set of size 1.
$\mathcal{G} \in \mathcal{F}_4$	$(*\text{-}\mathcal{J}_{strict}\text{-}*)$	$\Longleftrightarrow$	H_{strict} is a complete graph.
$\mathcal{G} \in \mathcal{F}_5$	$(1 - *)$	$\Longleftrightarrow$	G contains a dominating set of size 1.
$\mathcal{G} \in \mathcal{F}_6$	$(* - *)$	$\Longleftrightarrow$	G is a complete graph.
$\mathcal{G} \in \mathcal{F}_7$	$(*\text{-}\mathcal{J}\text{-}1)$	$\Longleftrightarrow$	H contains an in-dominating set of size 1.

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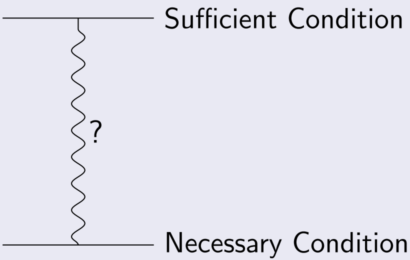

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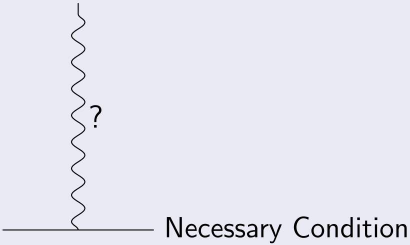

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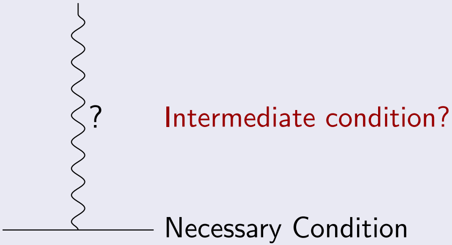

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- new resulting classes of evolving graphs
- some insights about the networking impact of mobility

Thank you

Questions?