Time-Varying Graphs and Dynamic Networks

–

(invited paper)

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ADHOC-NOW'11

Universität Paderborn

July 20th, 2011

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In fact, *highly* dynamic networks.

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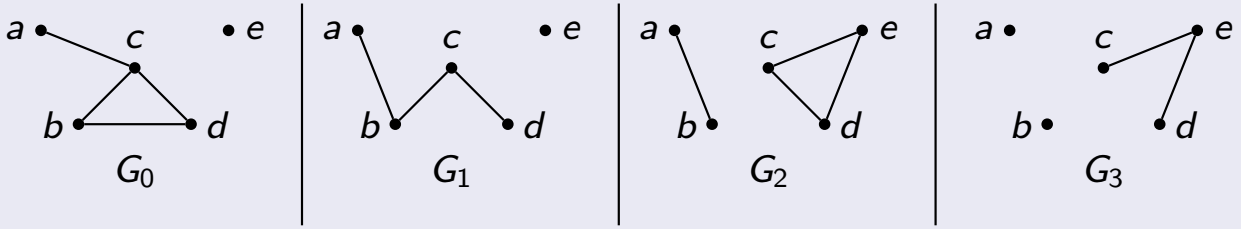
- Practical : evaluation through simulation
  → Reproducibility issues ; Limited understanding.
- Analytical : feasibility, correctness, complexity, *etc.*
  → Need for appropriate concepts & formalisms

Dynamic graphs

Evolving graphs [Ferreira 2004]

Sequence of graphs $\mathcal{G} = G_0, G_1, \ldots$



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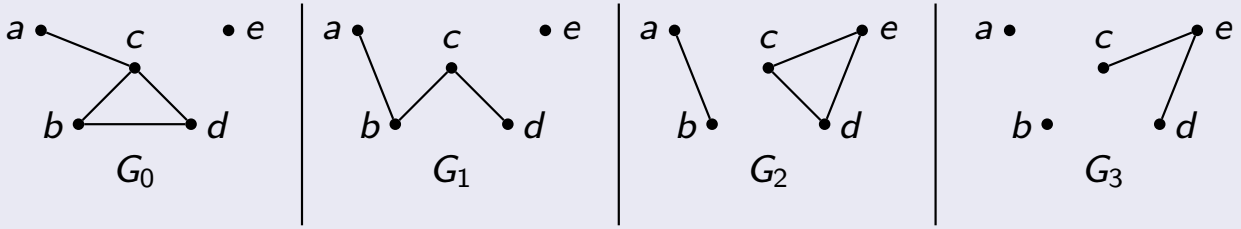

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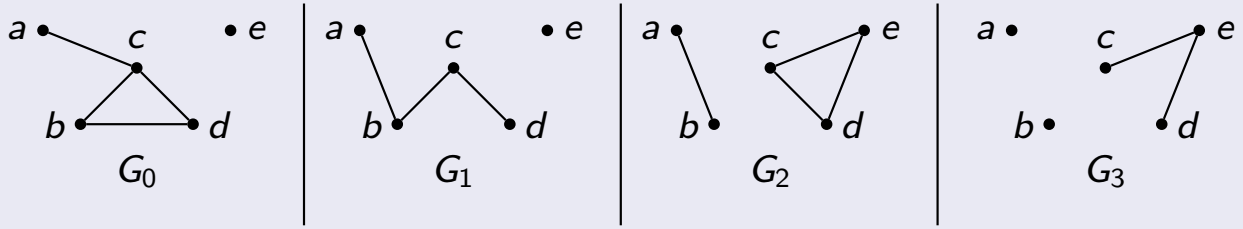

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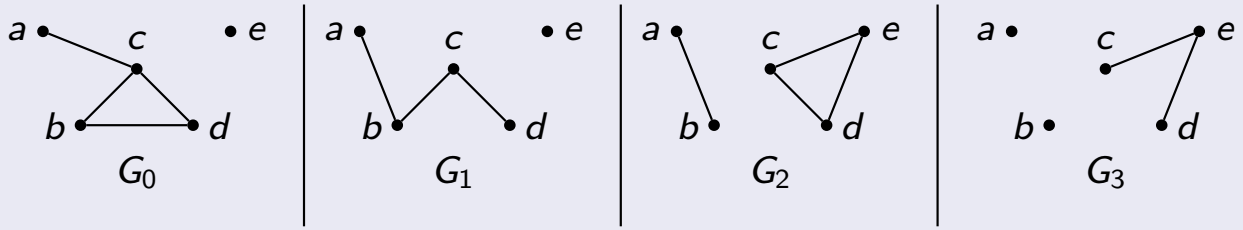

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$\mathcal{G} = (V, E, \quad)$

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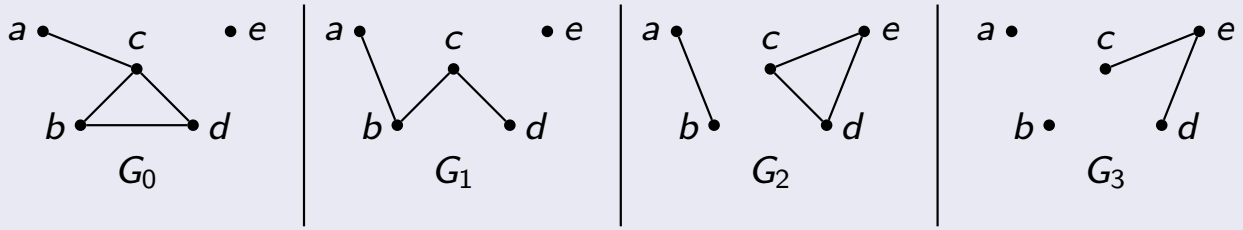

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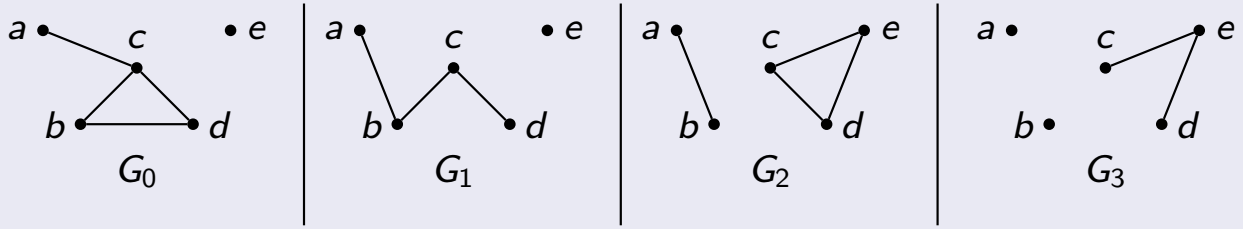

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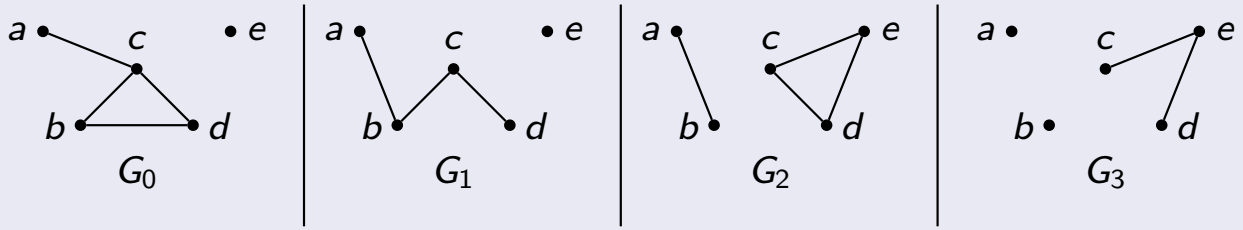

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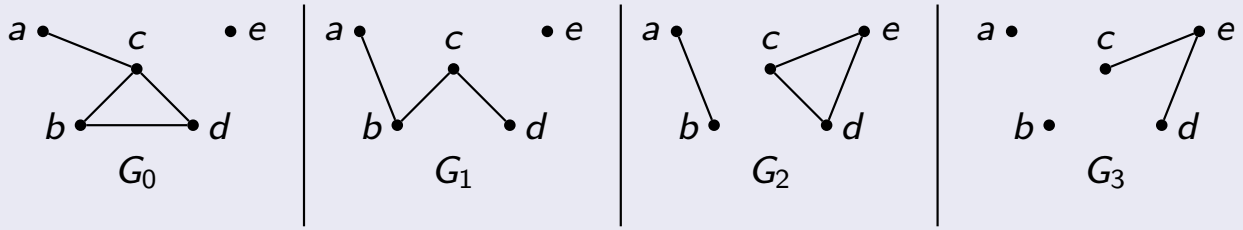

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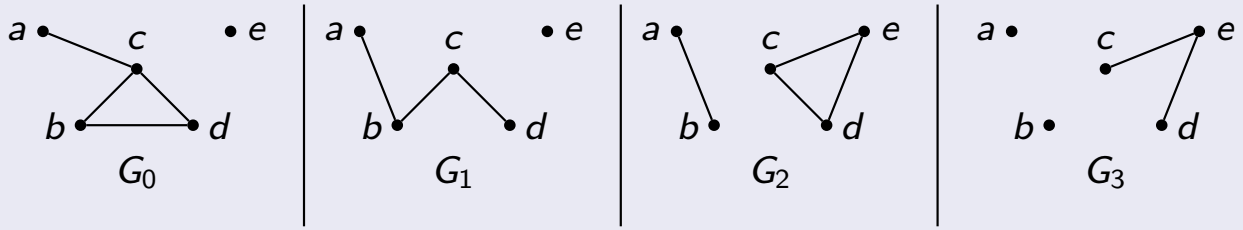

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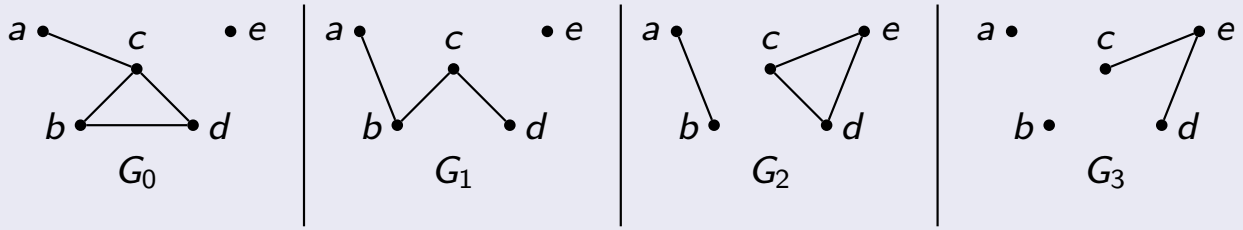

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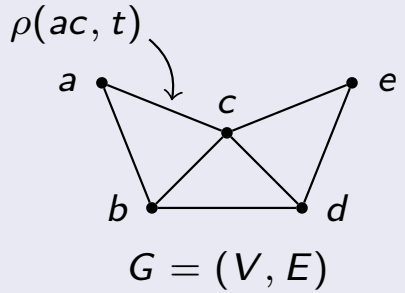

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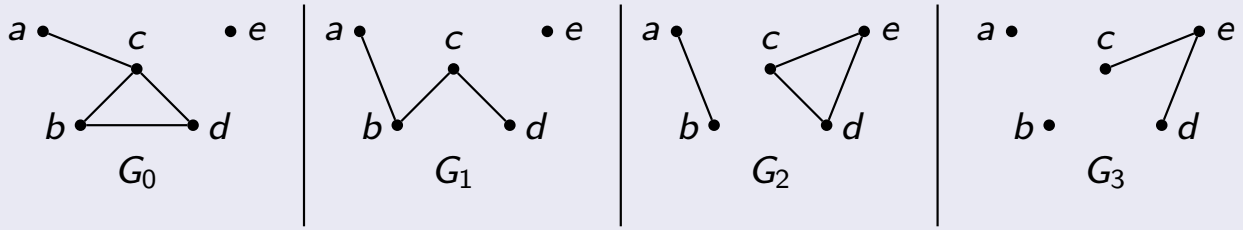

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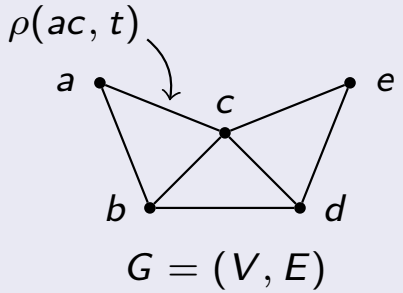

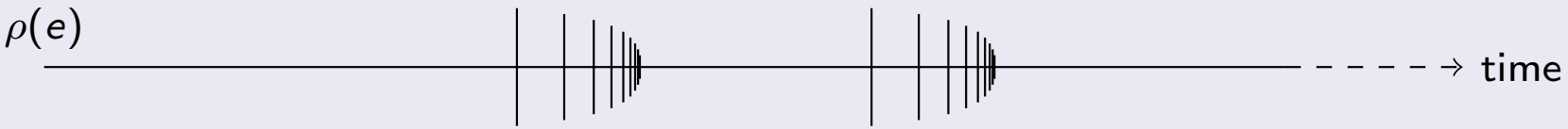

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Evolving graphs [Ferreira 2004] (*Graph-centric*)

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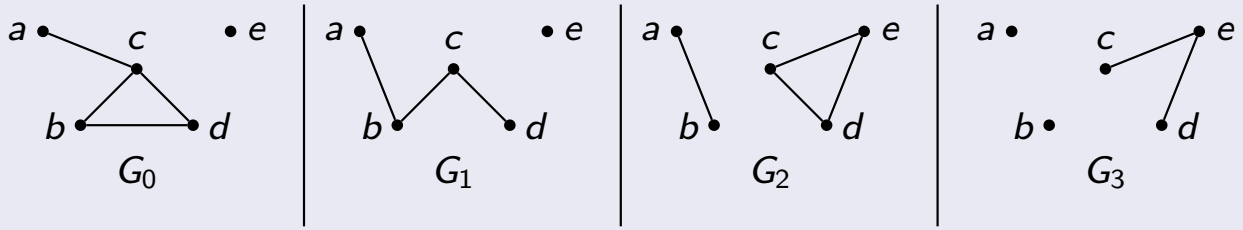

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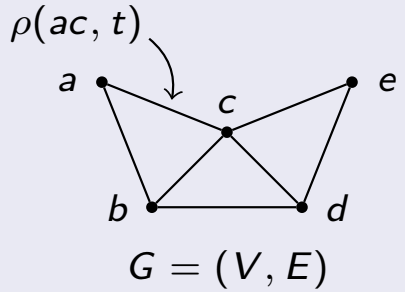
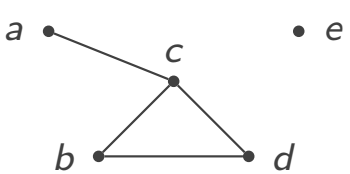
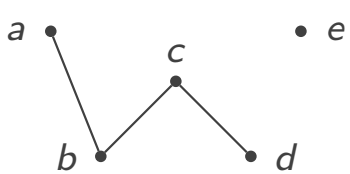


Mostly equivalent. We use them interchangeably based on the point of view.

What about graph concepts ?



G_0



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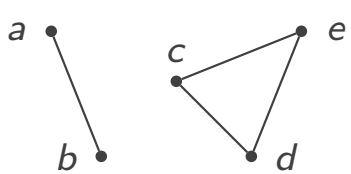
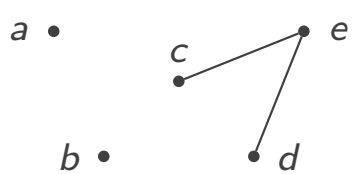


G_2



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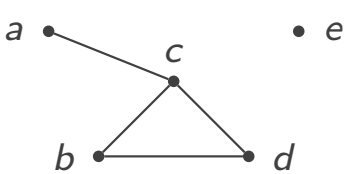

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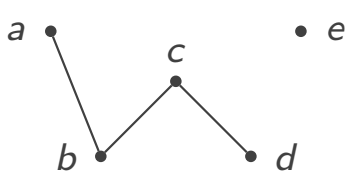

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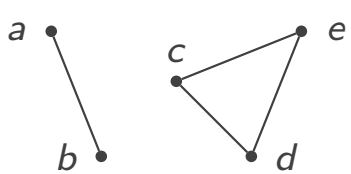

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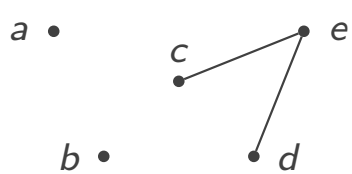

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$\Longrightarrow$ Connectivity over time.

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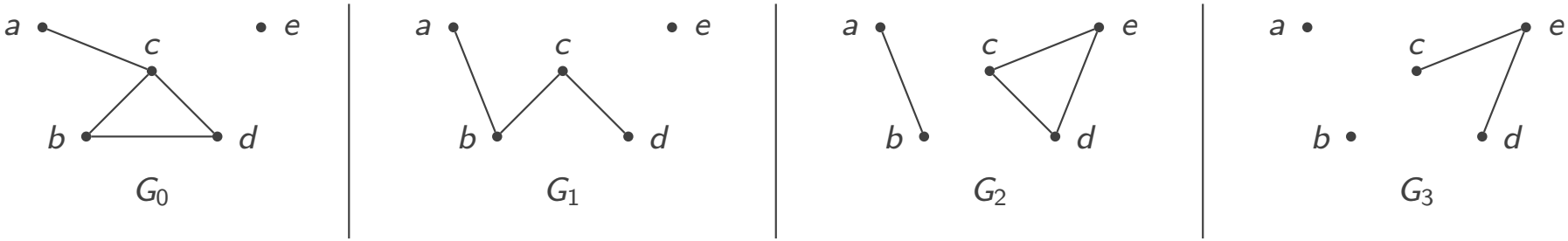

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Concept of *journey* : sequence of couples $\mathcal{J} = (e_1, t_1), (e_2, t_2) \ldots, (e_k, t_k)$, such that $e_1, e_2, ..., e_k$ is a walk in G, $t_1, t_2, ..., t_k$ is a non-decreasing sequence of dates from $\mathbb{T}$, and $\rho(e_i, t_i) = 1$ for all i.

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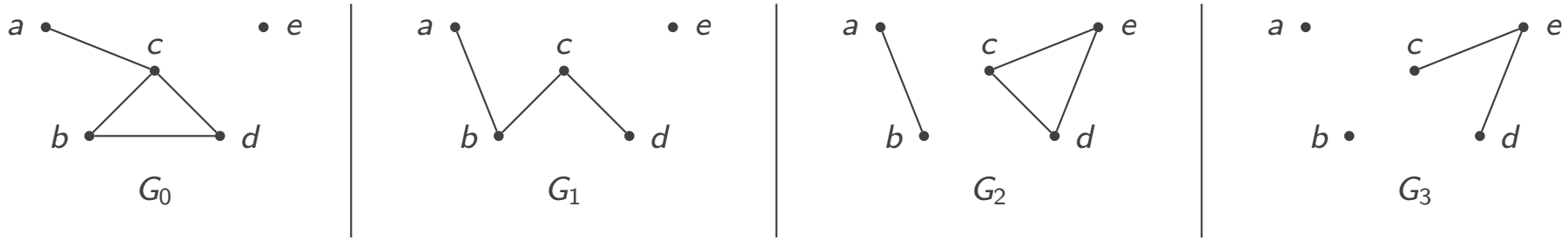

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Not symmetrical ! (e.g. $a \rightsquigarrow e$, but $e \not\rightsquigarrow a$)

Journeys in literature

- Schedule-conforming path [Berman, Networks, 1996]
- Time-respecting path [Kempe et al., STOC'00] [Holme, Physical Review, 2005]
- Temporal path [Chaintreau et al., Comm. Surveys & Tutorials, 2008]
- Journey [Bui-Xuan et al., J. Found. Comp. Sc., 2003]
- ...

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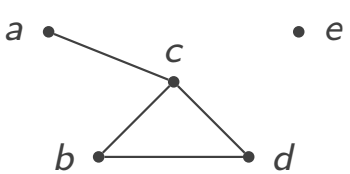

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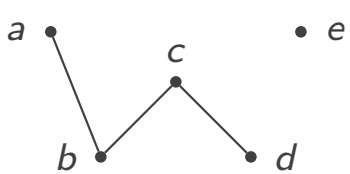

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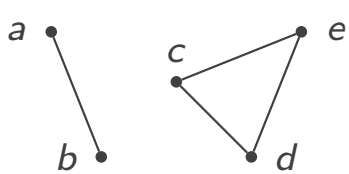

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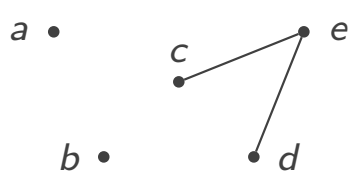

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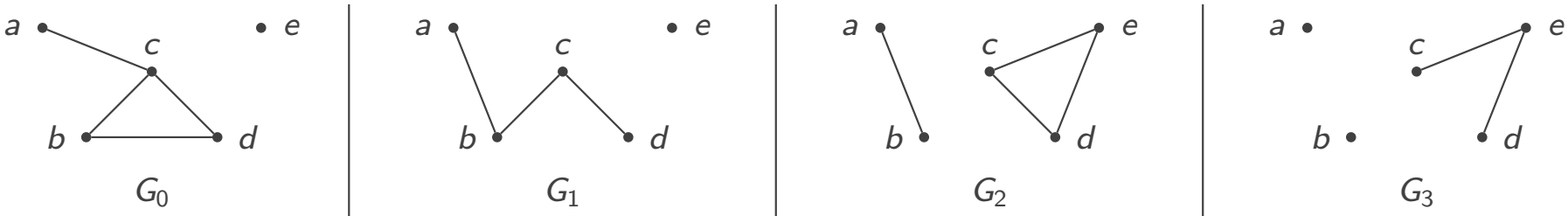

$\Longrightarrow$ Temporal distance.

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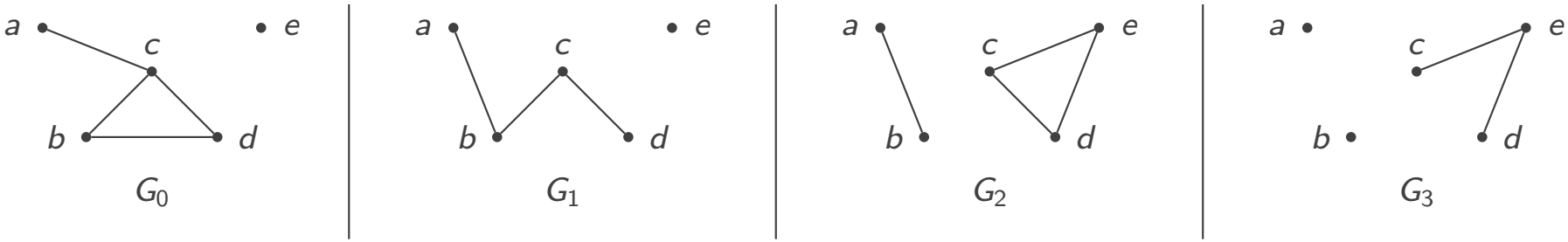

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Temporal distance in literature

- Minimal arrival date [Bui-Xuan et al., J. Found. Comp. Sc., 2003]
- Reachability time [Holme, Physical Review, 2005]
- Information latency [Kossinets et al., KDD'08]
- Temporal proximity [Kostakos, Physica A., 2009]
- ...

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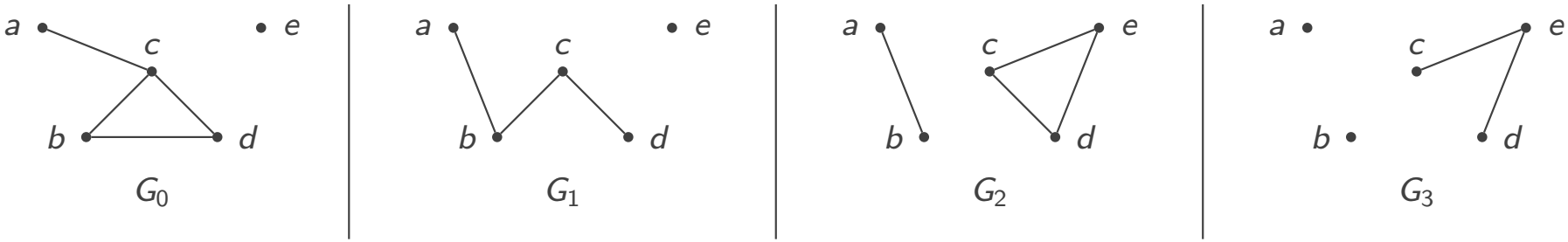

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Other concepts

- Time-Connected Components [Bhadra et al., ADHOC-NOW'03]
- Temporal diameter & eccentricity [Bui-Xuan et al., J. Found. Comp. Sc., 2003]
- Temporal View [Kossinets et al, KDD'08]
- Dynamic Expansion [Clementi et al, PODC'08]
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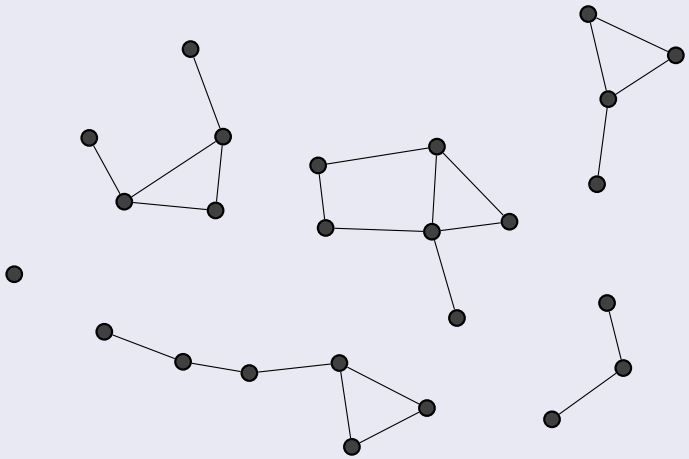
What about graph problems ?

"Classical" distributed problems (*e.g.* Election, SpanningTree, ...)



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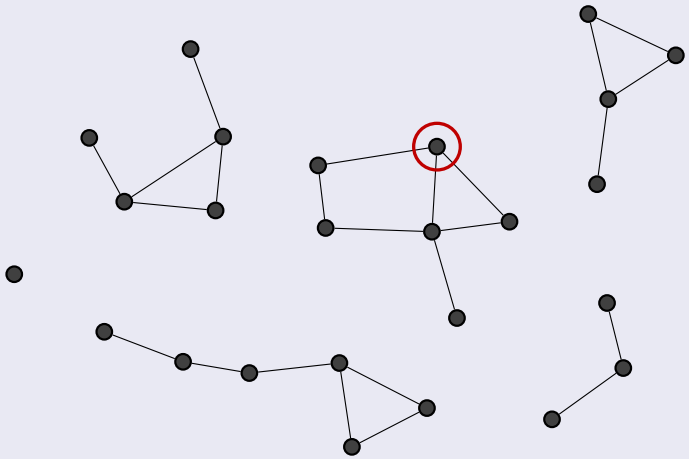

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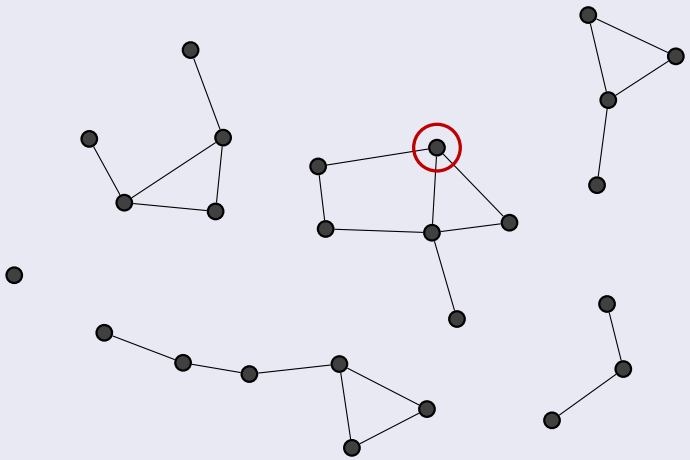

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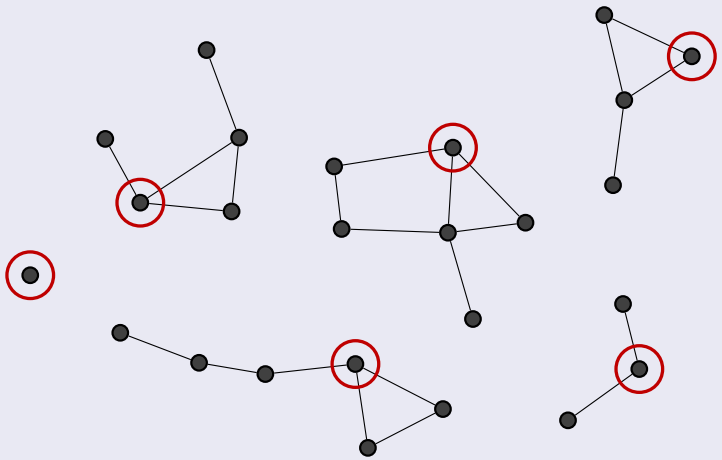

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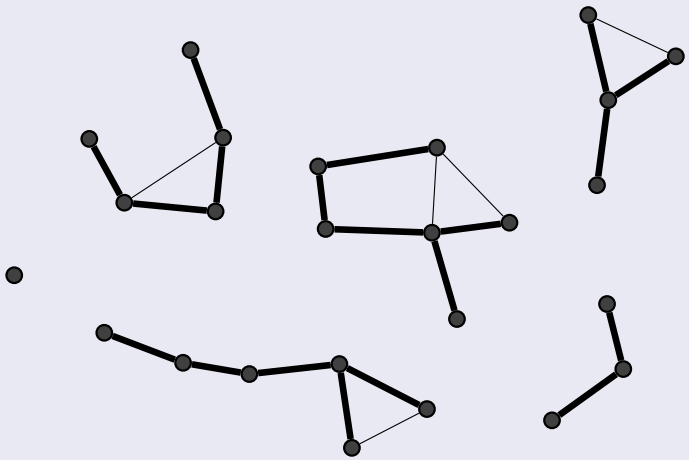

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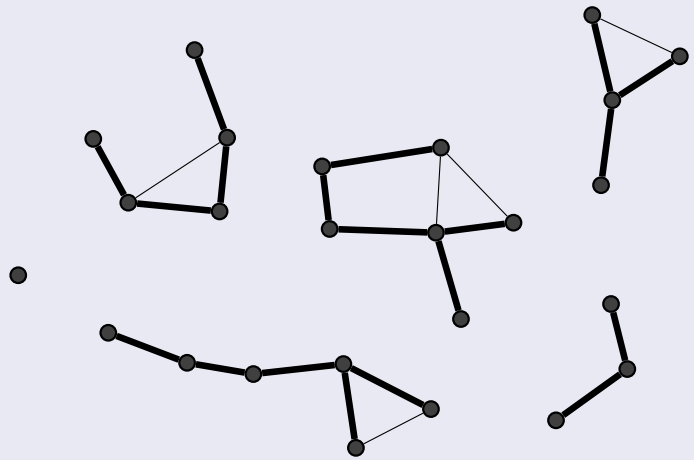

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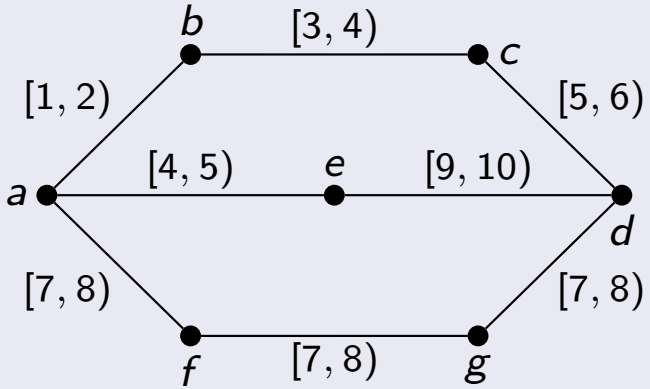
Optimal journey / broadcast [Bui-Xuan et al. 2003]



Several possible metrics !

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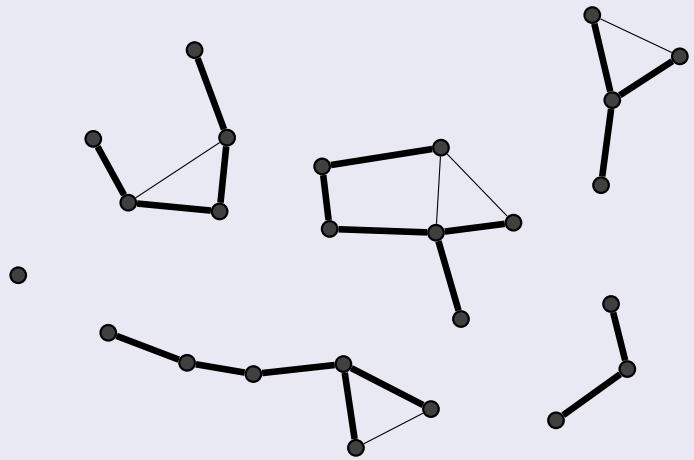

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→ Several options, e.g. :

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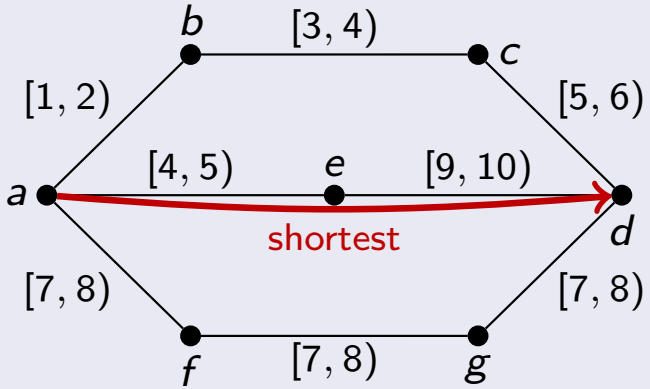
Optimal journey / broadcast [Bui-Xuan et al. 2003]



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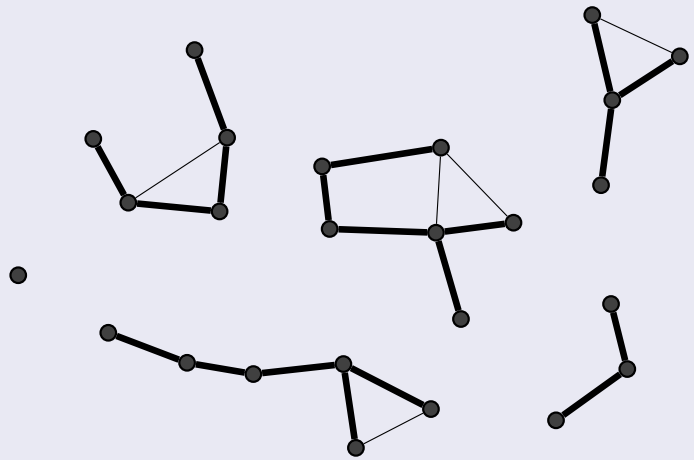

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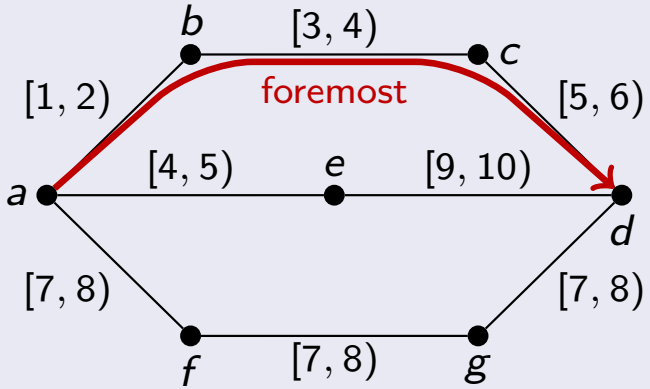

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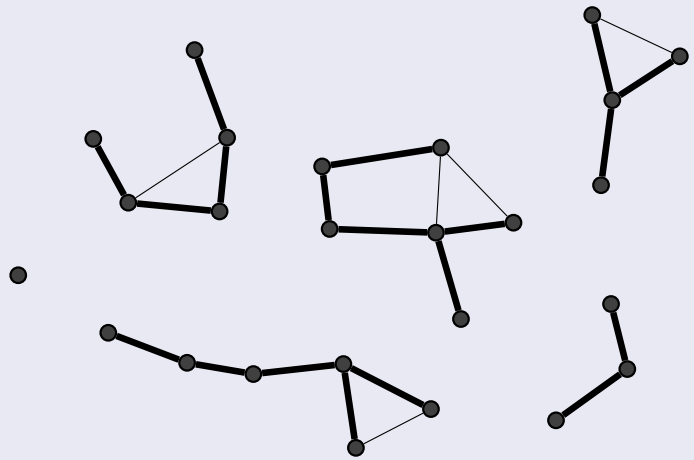

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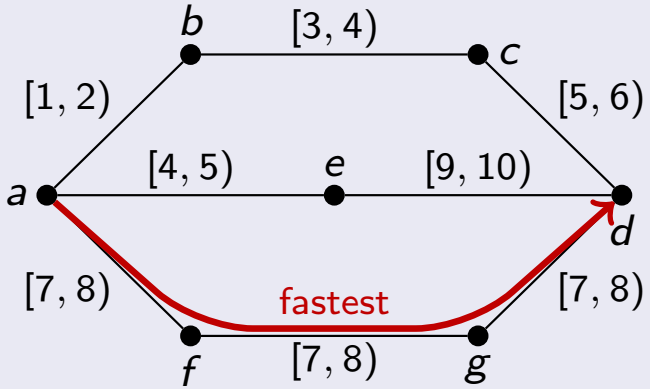

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Impact of properties on the feasibility/complexity ?

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Static properties on graphs

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Temporal properties on dynamic graphs ?

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- Recurrence of edges for SHORTEST BROADCAST (sufficient condition)

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$\rightarrow$ The field is just opening.

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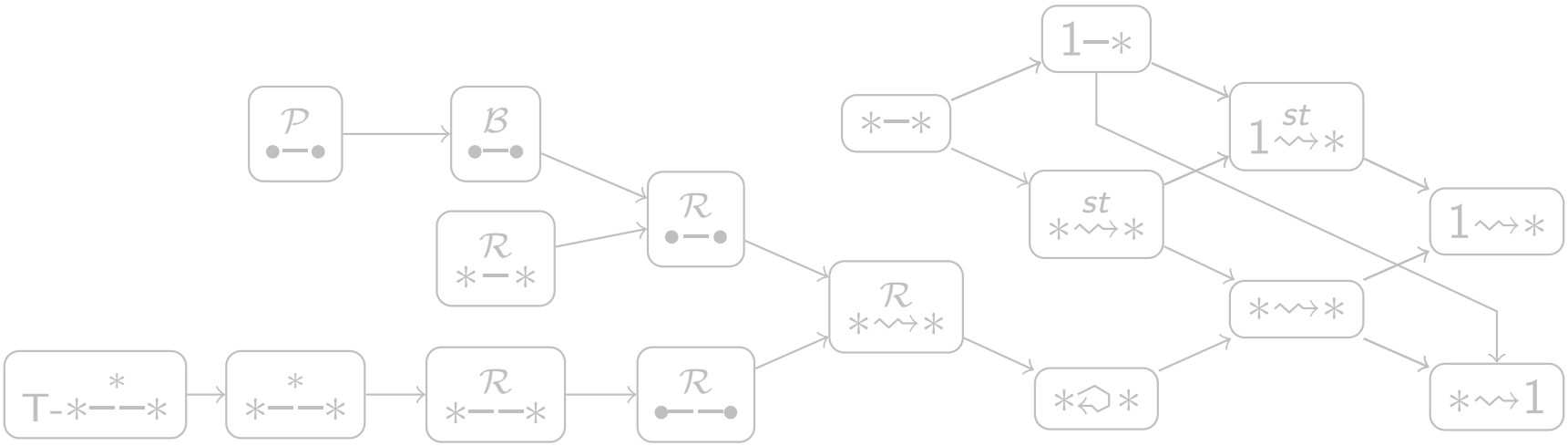
We are striving to collect and organize early results and emerging properties.

Some classes of dynamic graphs – hierarchy



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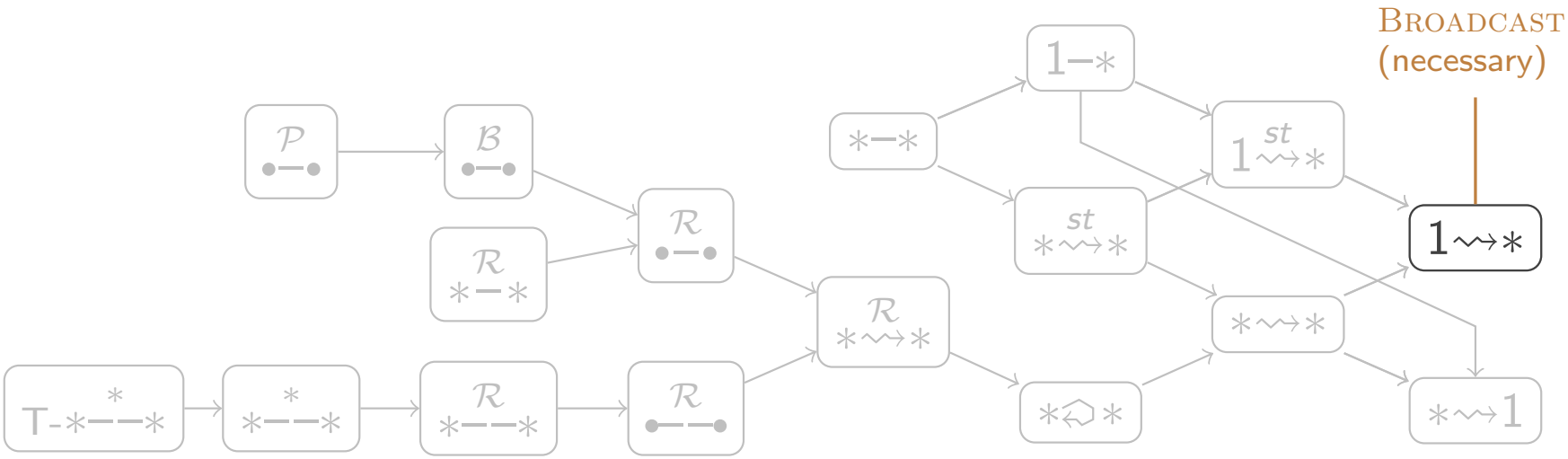

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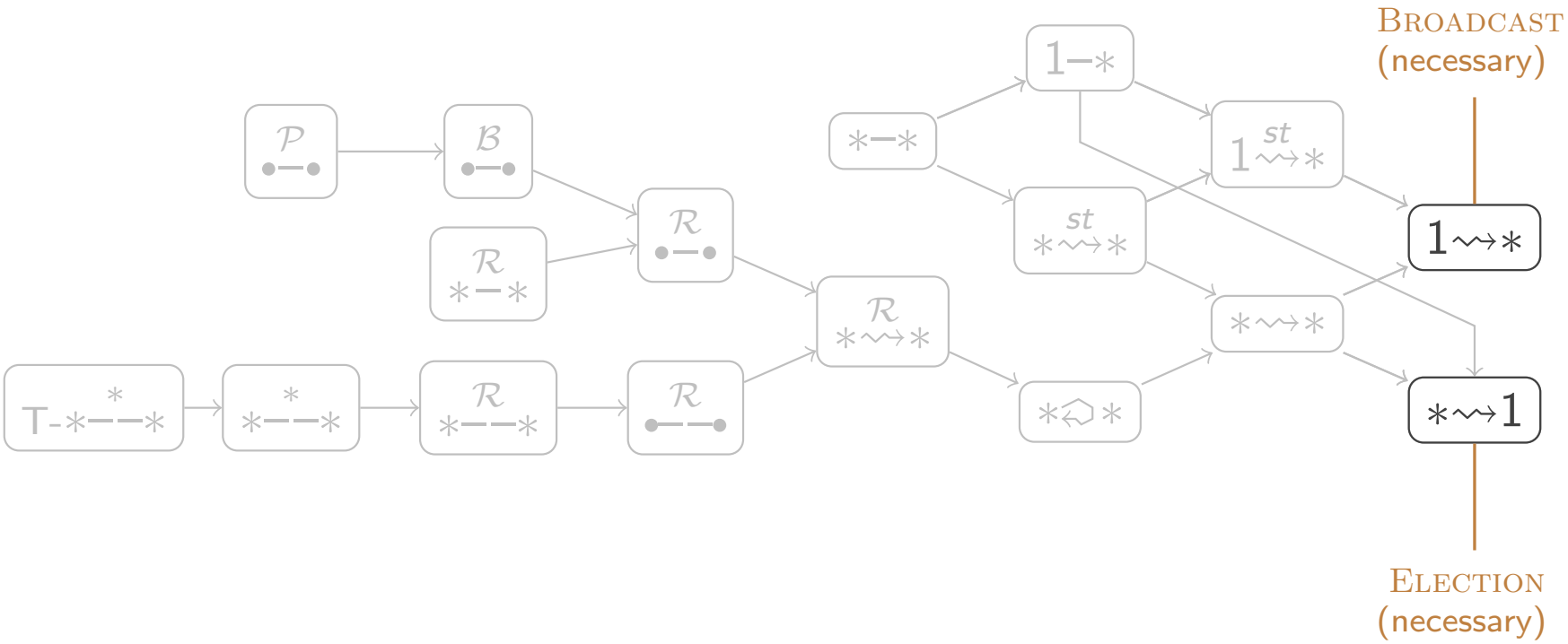

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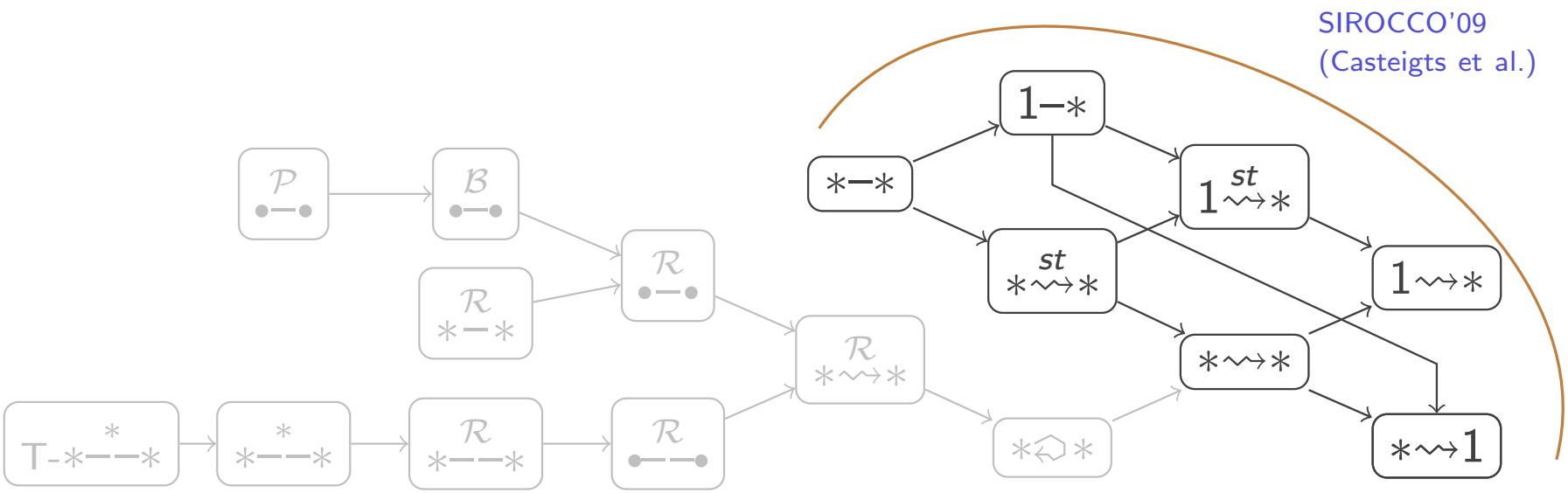

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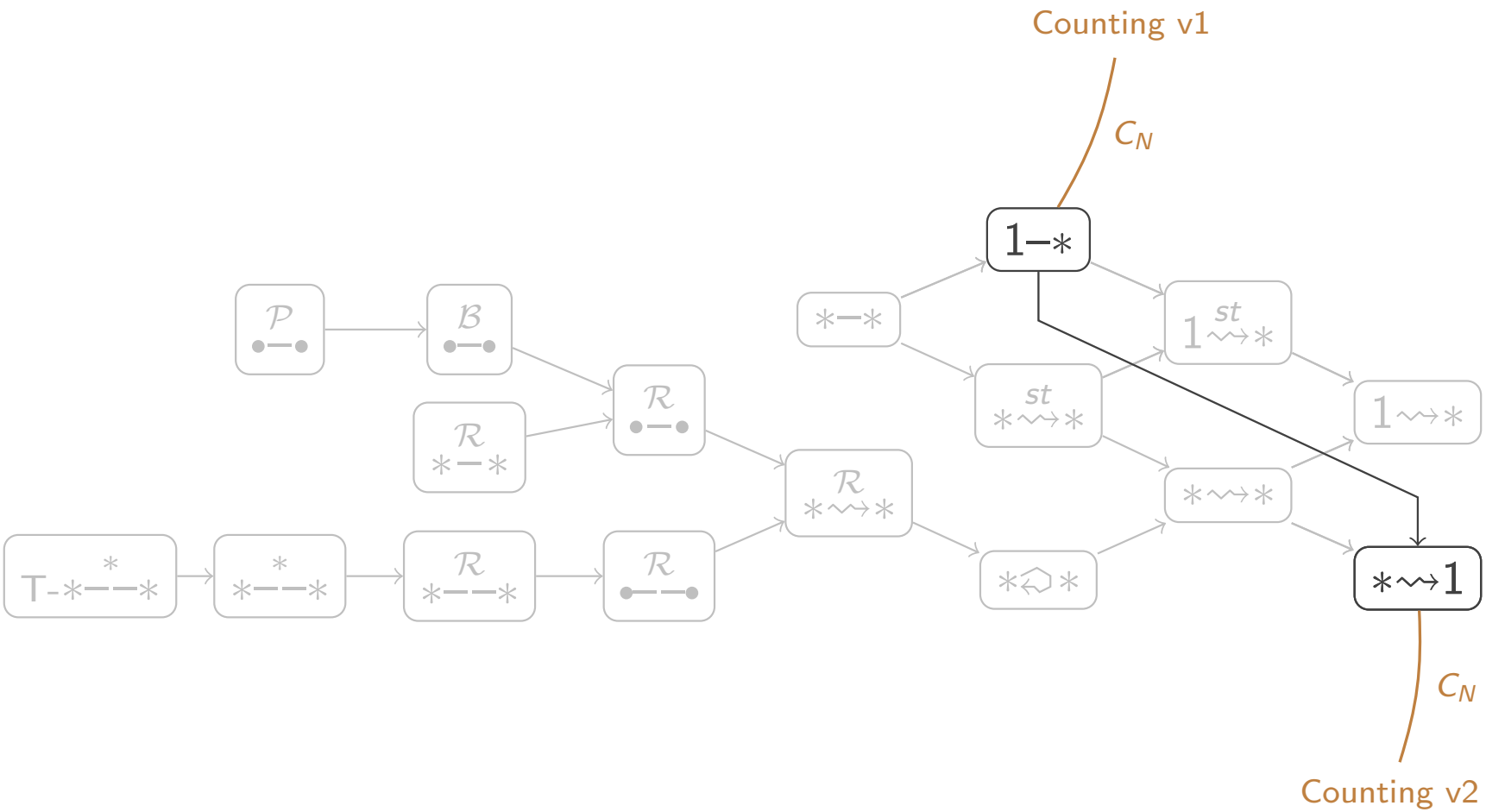

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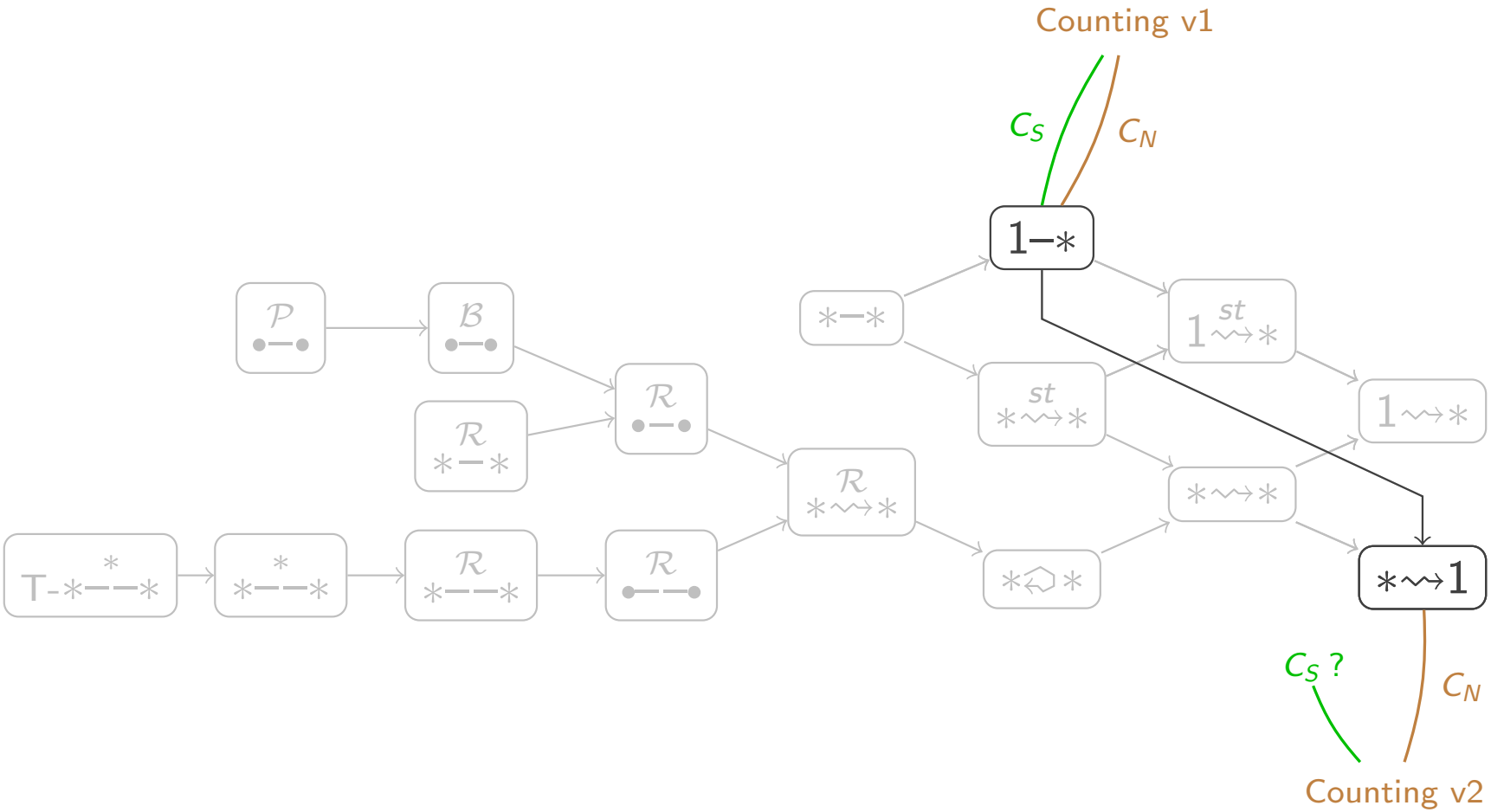

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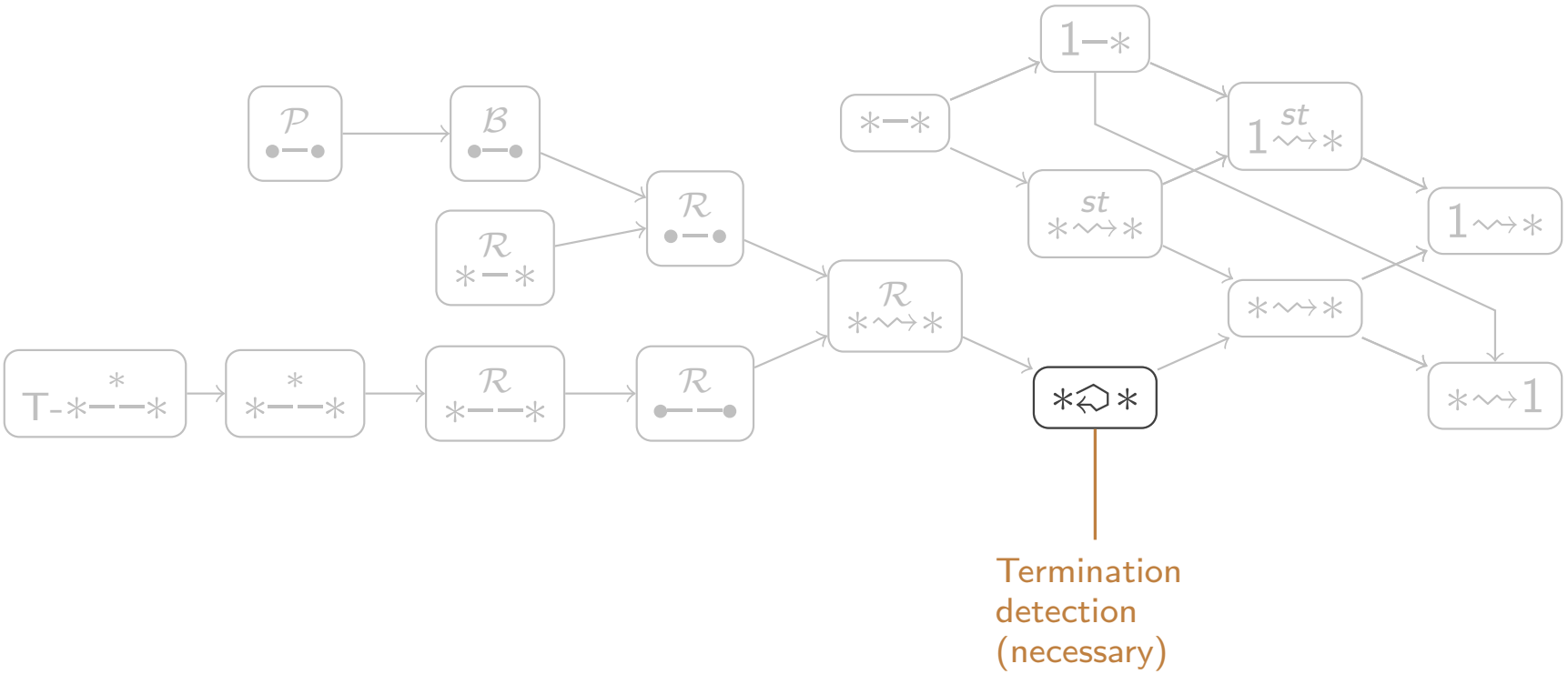

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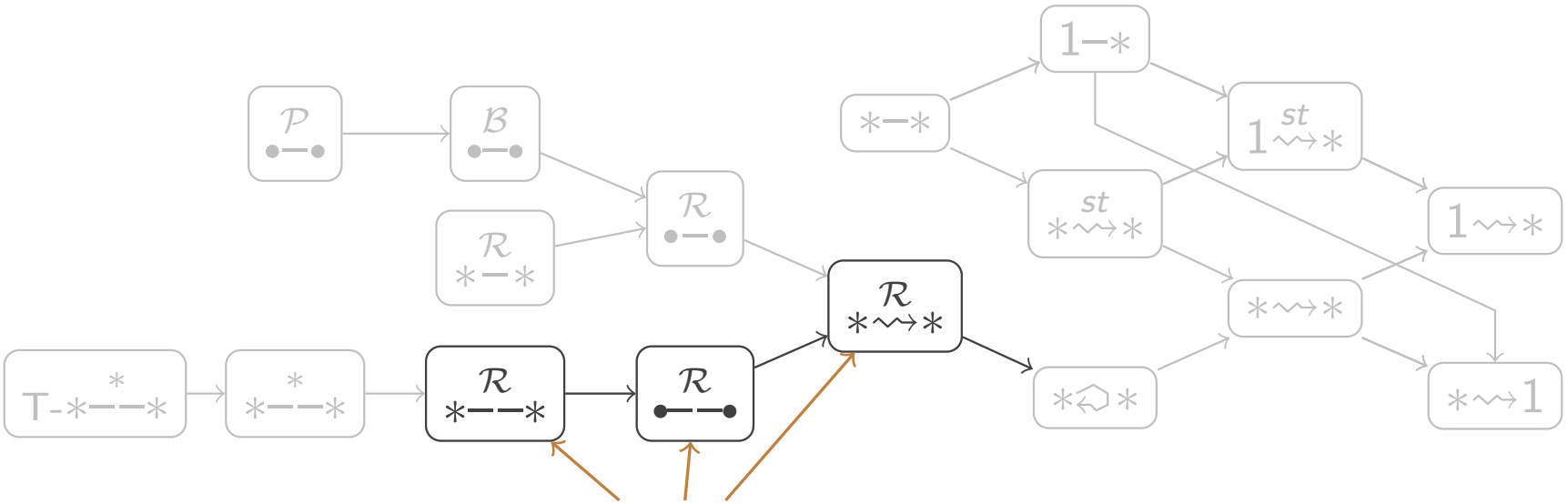

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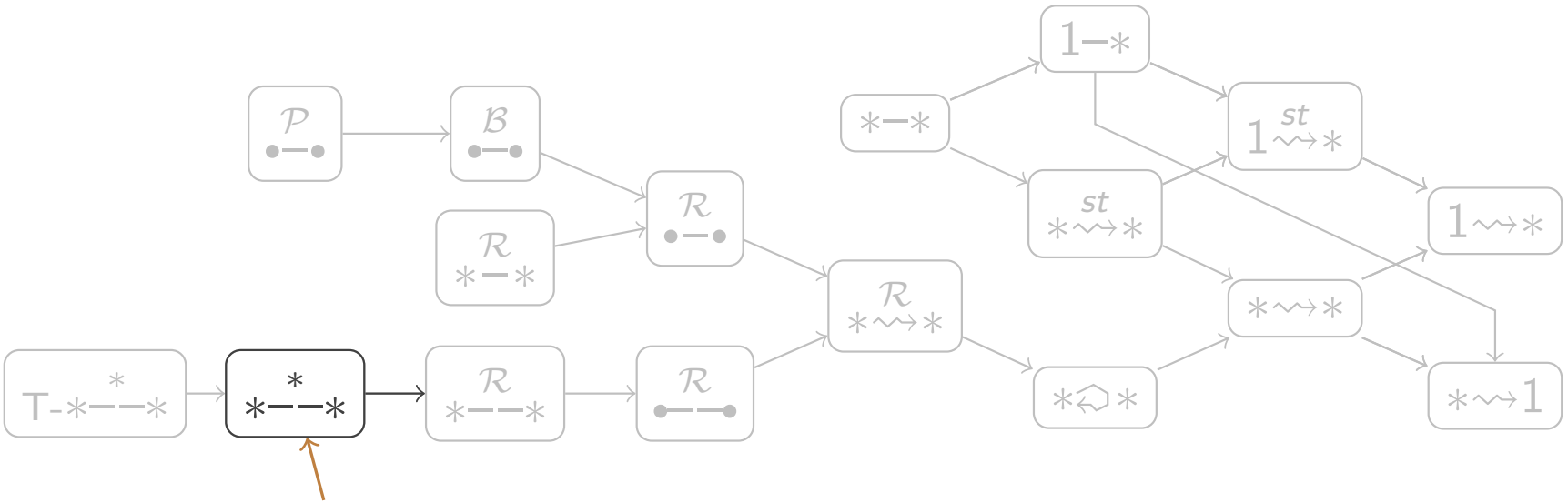

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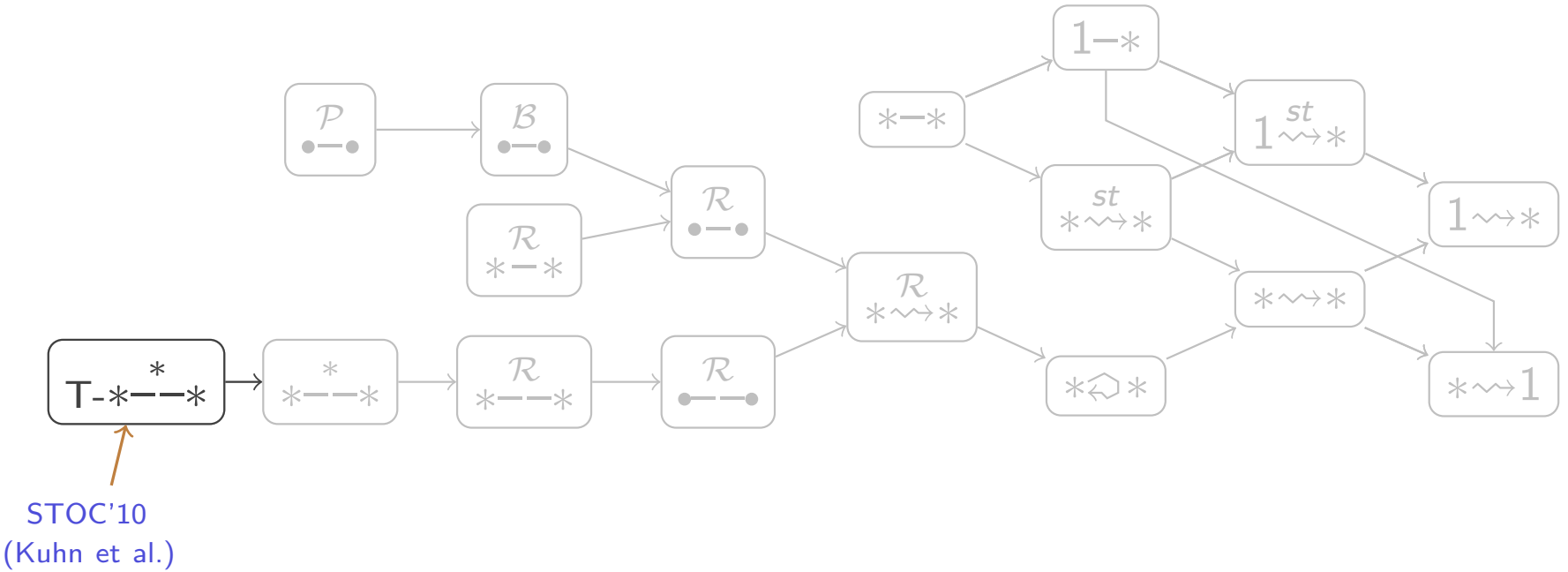

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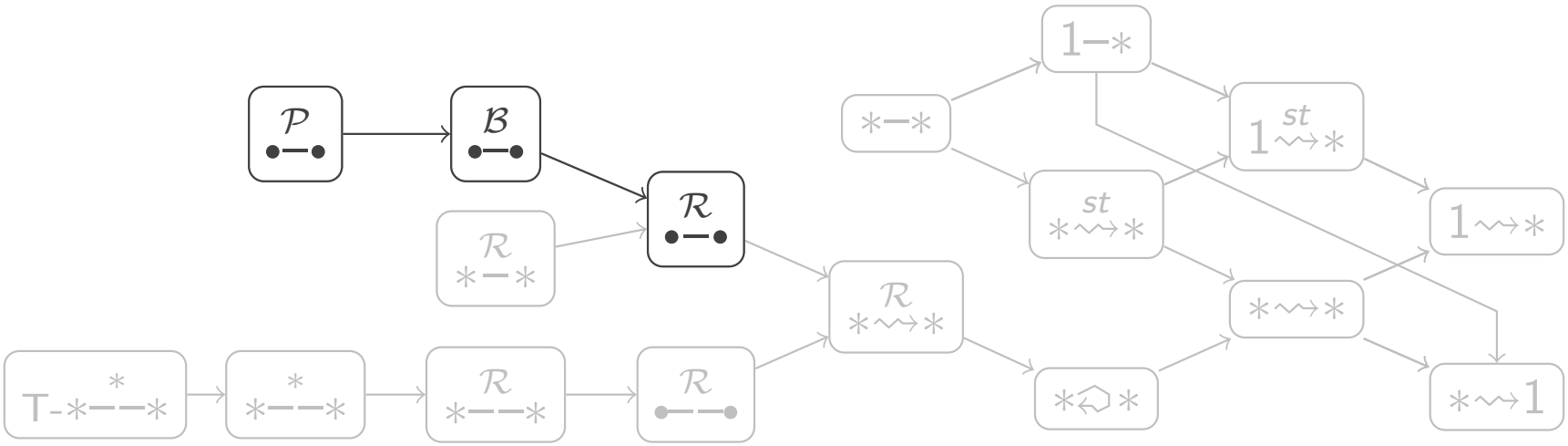

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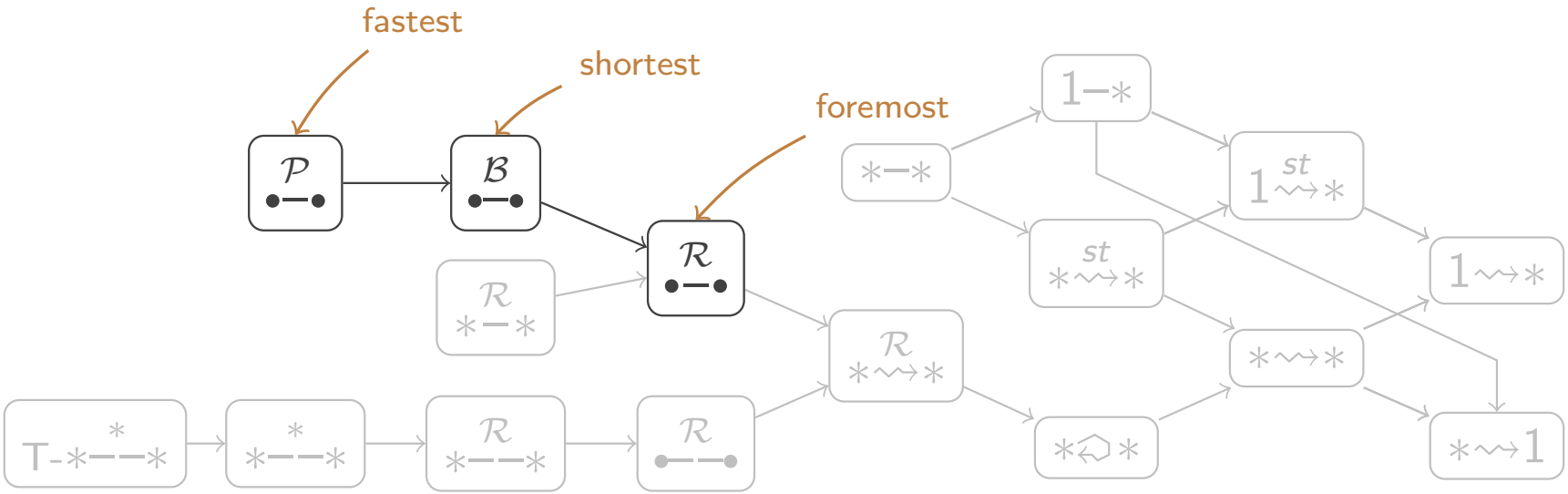

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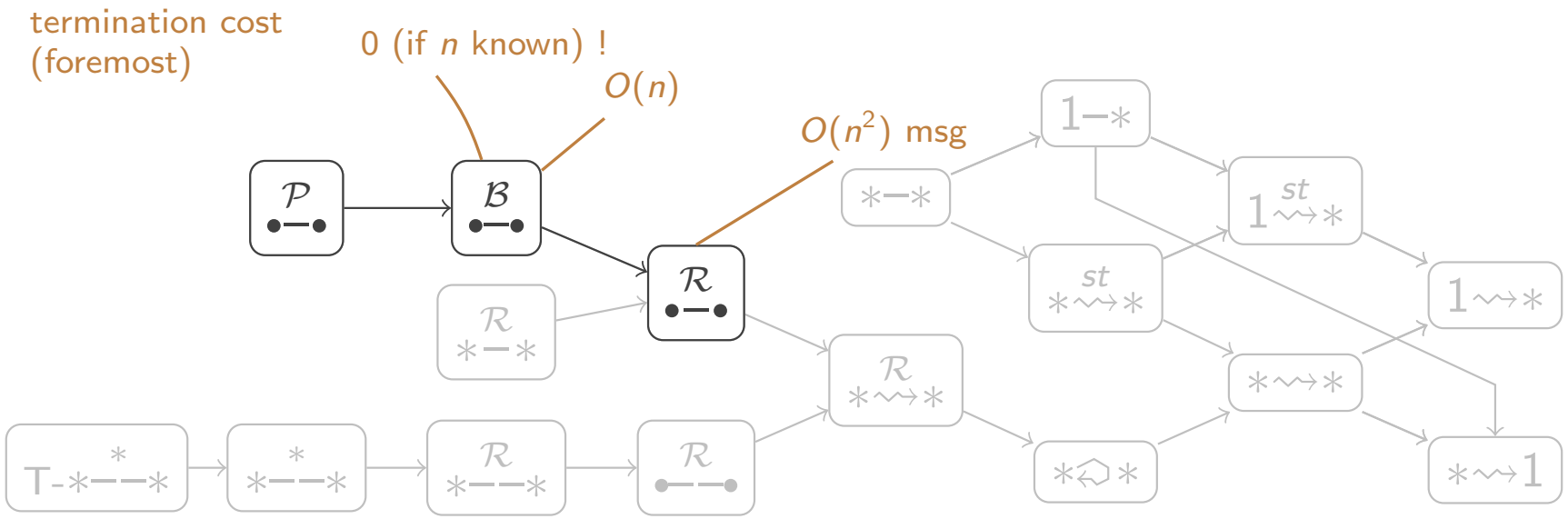

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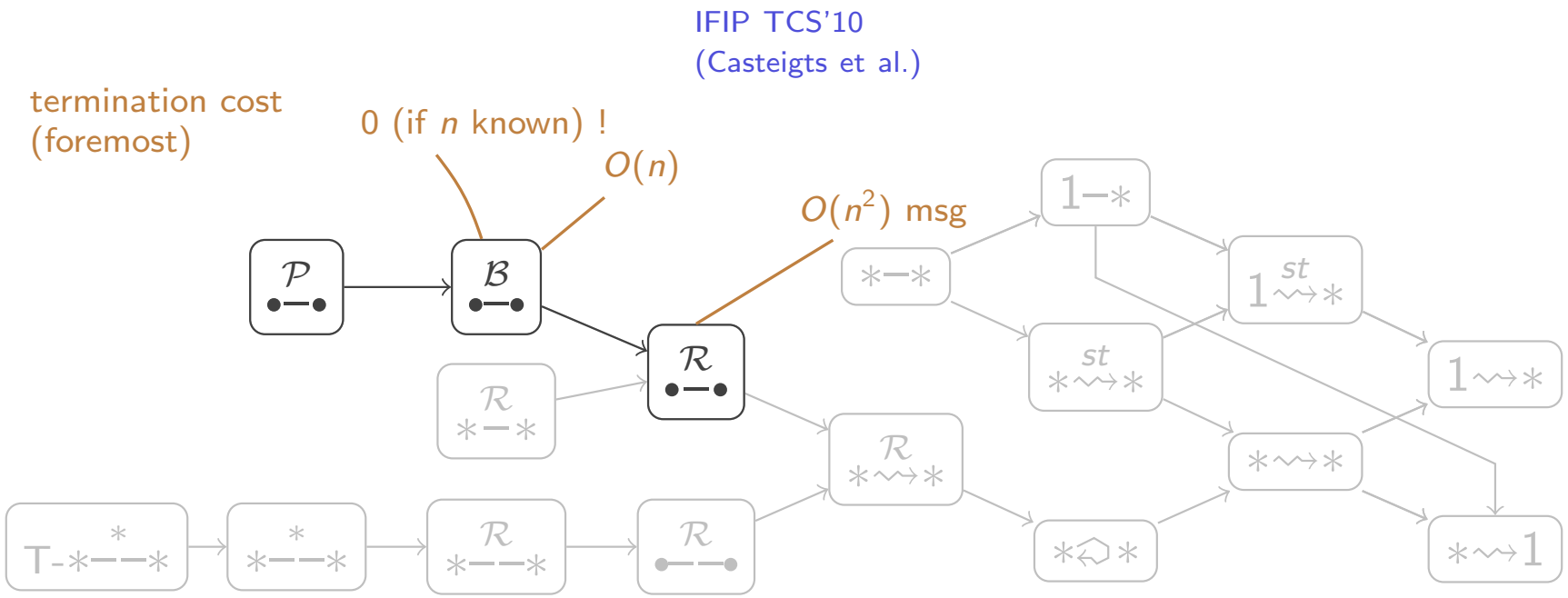

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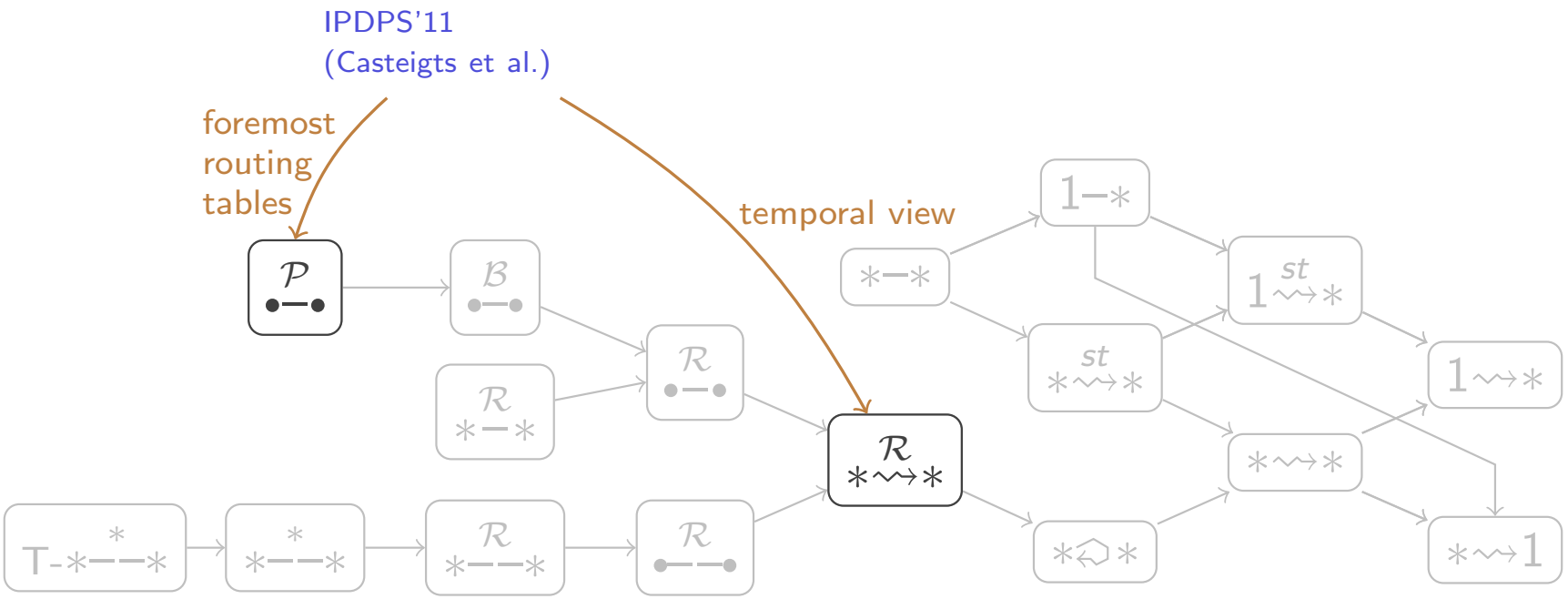

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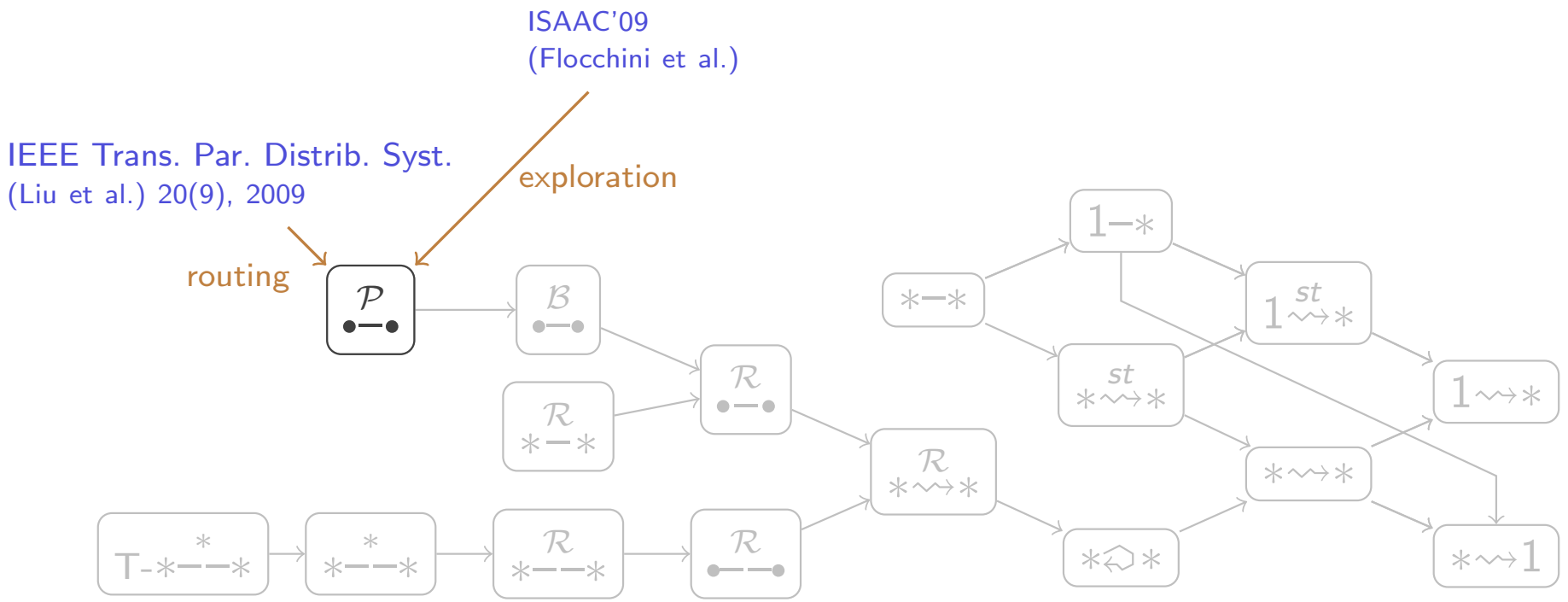

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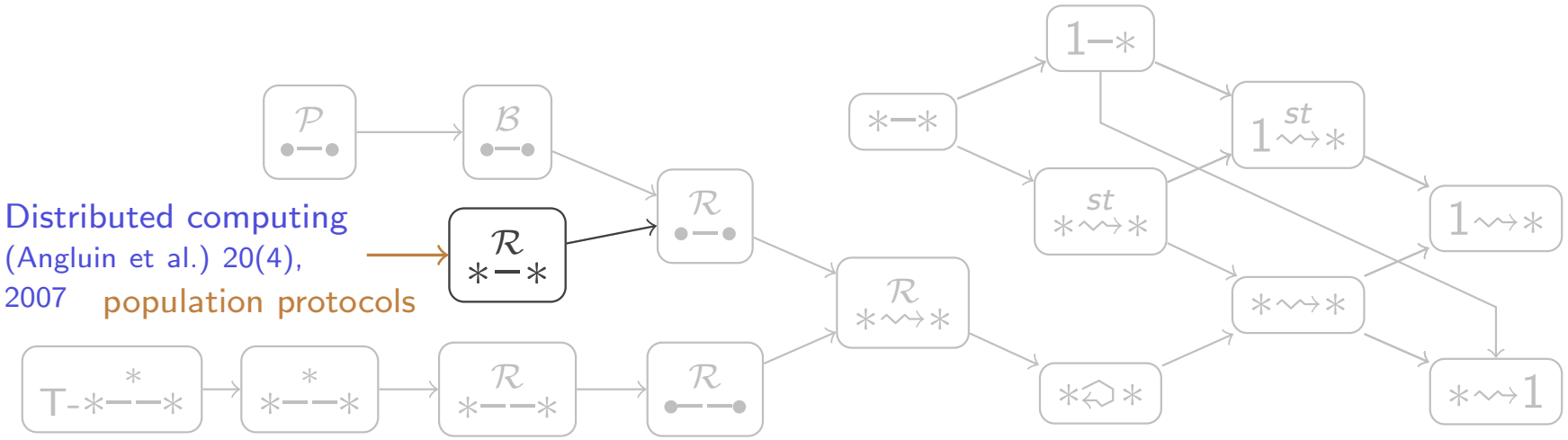

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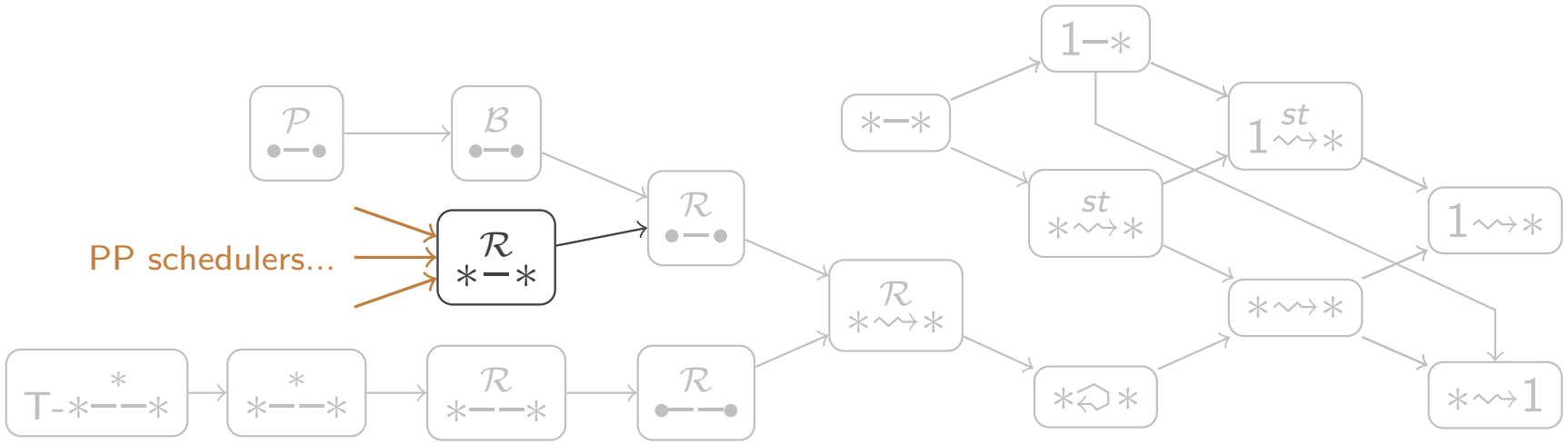

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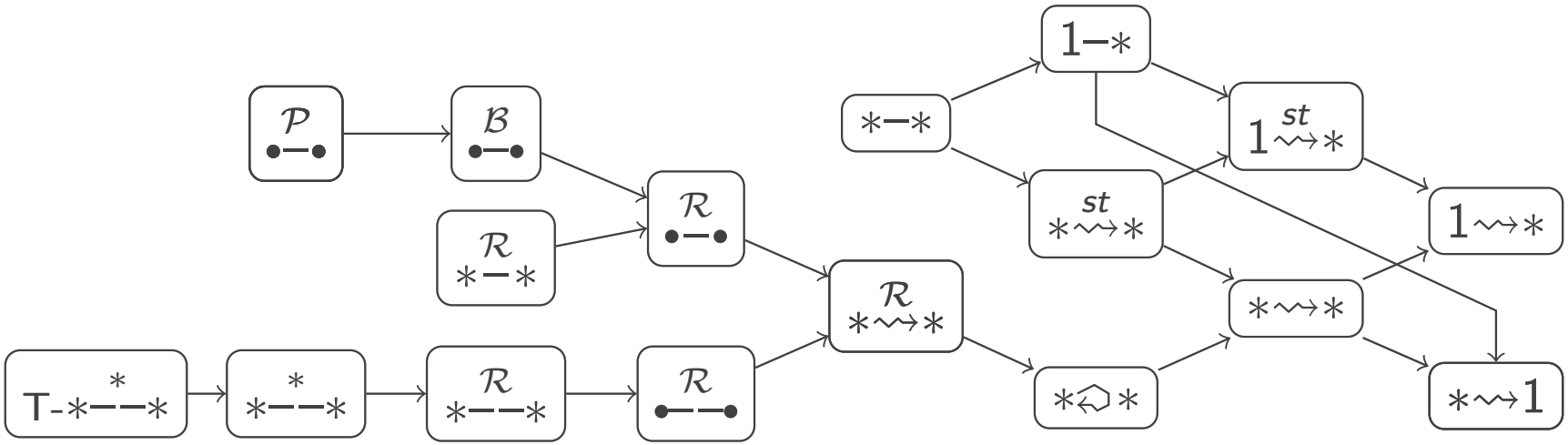

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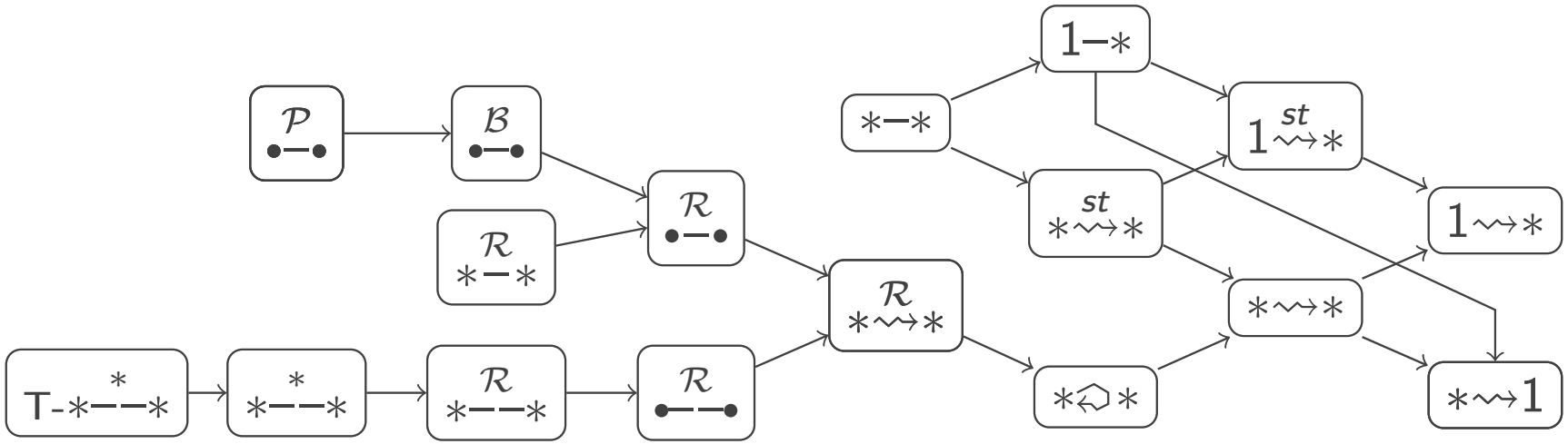

Thank you !