

# A Journey through Dynamic Networks (with Excursions)

Arnaud Casteigts

June 4, 2018

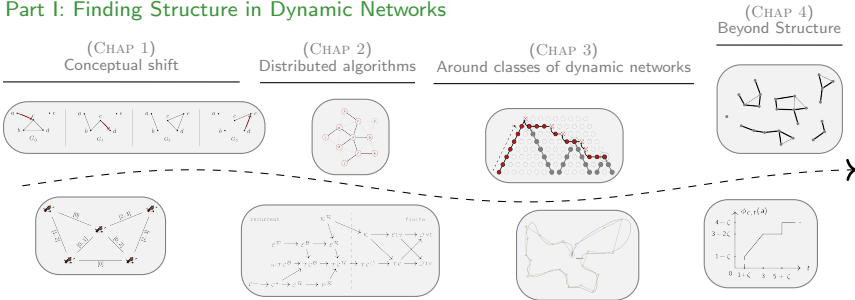
## Habilitation à diriger des recherches

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Invité	Joseph Peters	PU. Simon Fraser University (Vancouver)



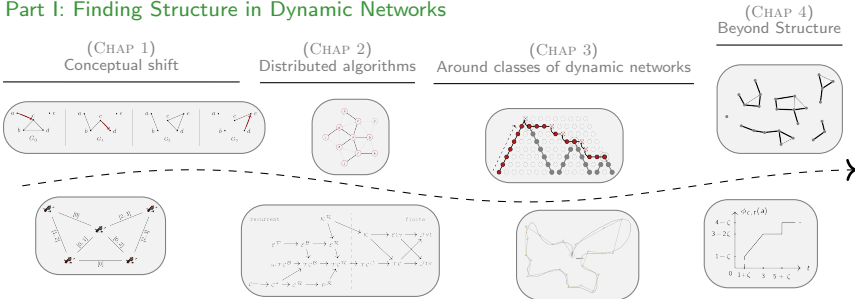
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## Part I: Finding Structure in Dynamic Networks

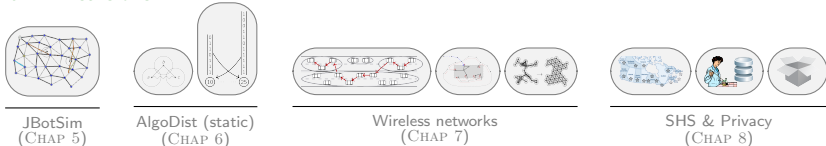


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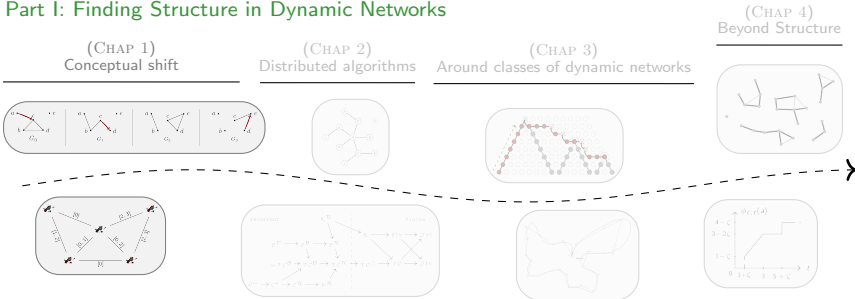


## Part II: Excursions

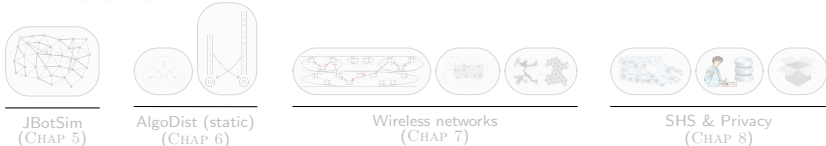


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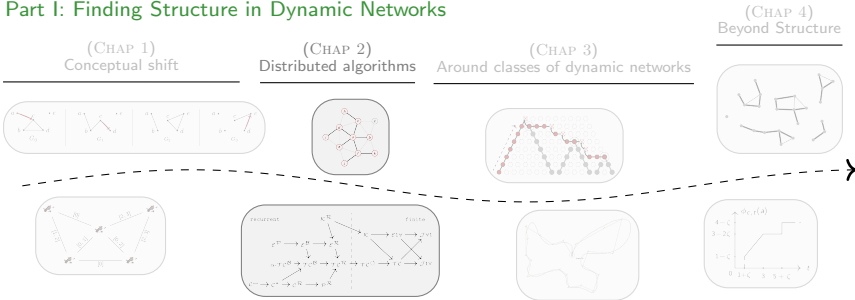


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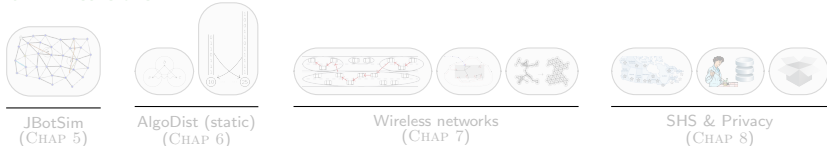


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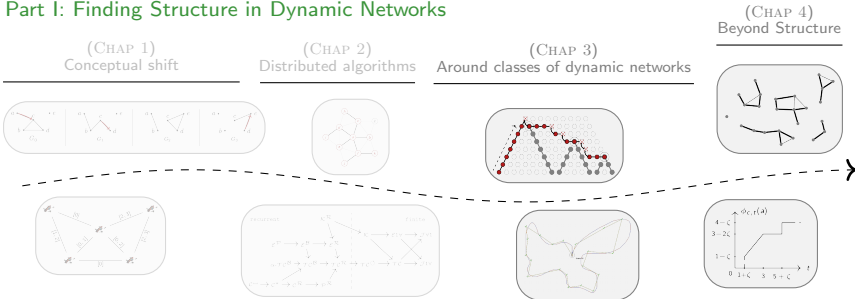


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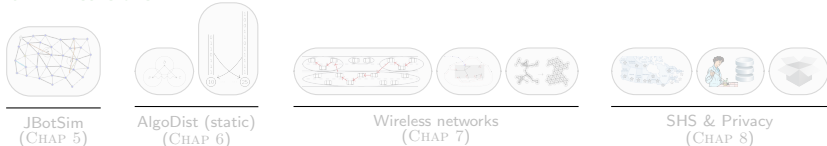


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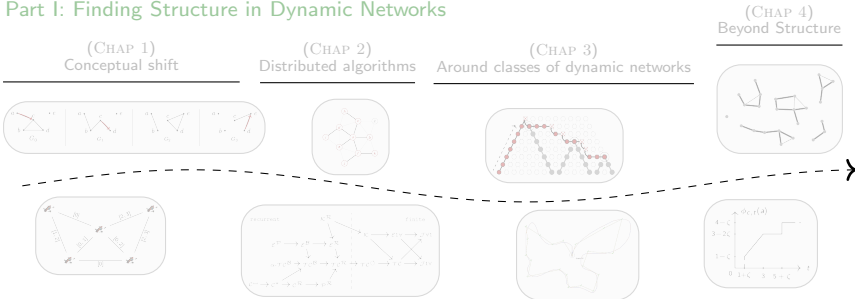


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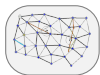


# A Journey through Dynamic Networks (with Excursions)

## Part I: Finding Structure in Dynamic Networks



## Part II: Excursions



JBotSim  
(CHAP 5)



AlgoDist (static)  
(CHAP 6)



Wireless networks  
(CHAP 7)

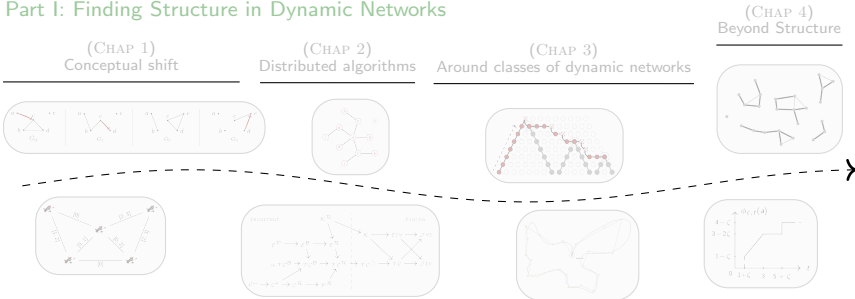


SHS & Privacy  
(CHAP 8)

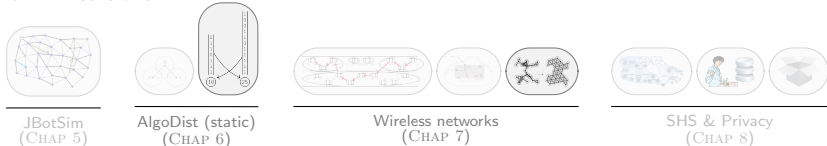


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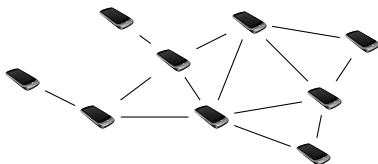




# Introduction

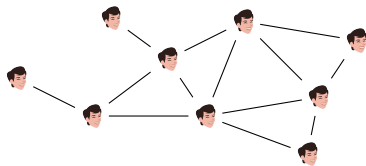
# Networks?

- Set of **nodes**  $V$  (a.k.a. entities, vertices)
  - Set of **links**  $E$  among them (a.k.a. relations, edges)
- A **network** (or graph)  $G = (V, E)$



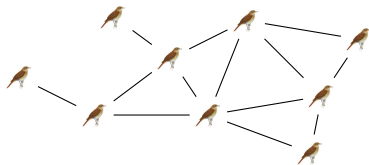
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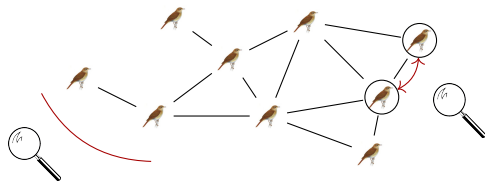
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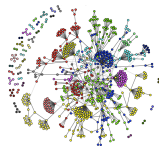
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## Complex networks

- compute global metrics
- explain and reproduce phenomena



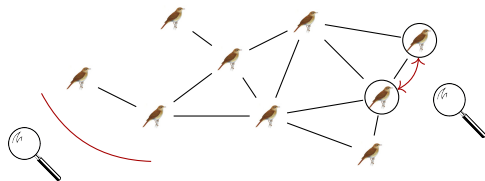
## Communication networks

- design interactions among entities
- study what can be done *from within*



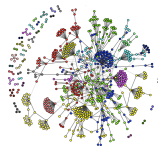
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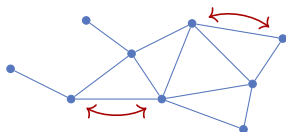
## Communication networks

- design interactions among entities
- study what can be done *from within*
- distributed algorithms...



# Distributed Algorithms

(Think globally, act locally)

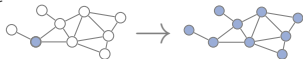


Collaboration of distinct entities to perform a common task.

No centralization available.

## Examples of problems:

Broadcast



Election



Spanning tree



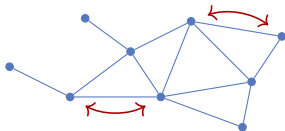
Counting



Consensus, naming, routing, exploration, dominating sets, ...

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## Leader Election (distributed problem)

→ Distinguishing one node among all

Version with ID → highest ID elected



	Time	# Messages	Message size	
Awerbuch'87	$O(n)$	$\Theta(m + n \log n)$	$O(\log n)$ bits	
Peleg'90	$\Theta(D)$	$O(Dm)$	$O(\log n)$ bits	

Optimal in time and number of messages?

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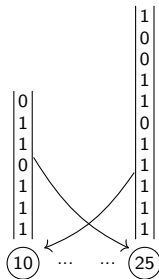
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Peleg'90	$\Theta(D)$	$O(Dm)$	$O(\log n)$ bits	$O(D \log n)$
<i>Our contrib.</i>	$O(D + \log n)$	$O((D + \log n)m)$	$O(1)$ bits	$\Theta(D + \log n)$

Optimal in time and number of messages?Yes! With **constant size** messages →  $O(D + \log n)$  **bit rounds** algorithm+ matching lower bound:  $\Omega(D + \log n)$ 

## The algorithm

Bitwise dissemination of highest ID defining a spanning tree

New encoding technique for IDs

→ ex:  $ID = 25 \stackrel{2}{=} 11001$ , then  $\alpha(ID) = 11111011001$ 

## Dynamic networks?



How to approach these contexts?

Applied *versus* theoretical?

## Dynamic networks?



Applied *versus* theoretical?

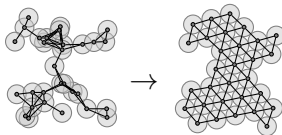
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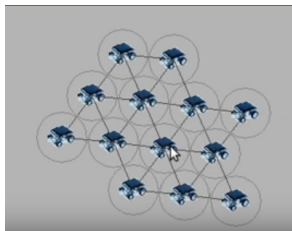
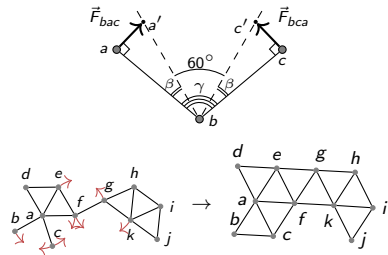
# Excursion: Biconnecting robots with virtual angular forces (CHAP 7 SEC 3)

Problem: Deploying robots from arbitrary connected configuration, with consideration to

- ▶ Coverage (max)
- ▶ Movements (min)
- ▶ Diameter (min)
- ▶ Biconnectivity (fault tolerance)



Approach: spring forces (attraction/repulsion) + angular forces



# Dynamic networks?



Applied *versus* theoretical

How to approach these contexts?

# Dynamic networks?

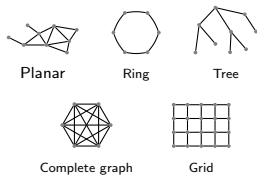


How to approach these contexts?

Applied *versus* theoretical

→ Working with structure (mostly theoretical)

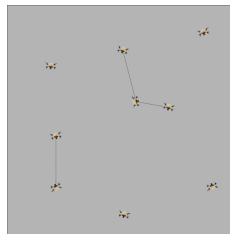
## Static networks



... can be exploited by an algorithm

## (Highly) dynamic networks

What kind of structure?

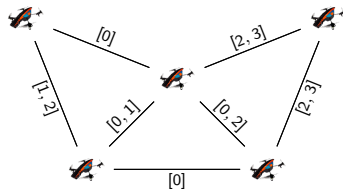




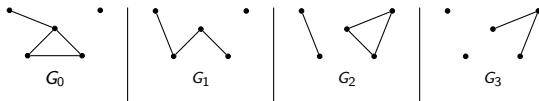
## Time-varying graphs (TVG)

$$\mathcal{G} = (V, E, \mathcal{T}, \rho, \zeta)$$

- $\mathcal{T} \subseteq \mathbb{N}/\mathbb{R}$  (lifetime)
- $\rho : E \times \mathcal{T} \rightarrow \{0, 1\}$  (presence function)
- $\zeta : E \times \mathcal{T} \rightarrow \mathbb{N}/\mathbb{R}$  (latency function)



Another classical view  $\mathcal{G} = G_0, G_1, \dots$



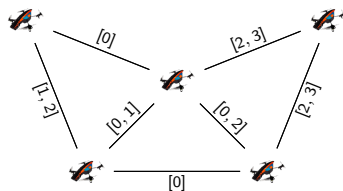
Variety of models and terminologies:

Dynamic graphs, evolving graphs, temporal graphs, link streams, etc.

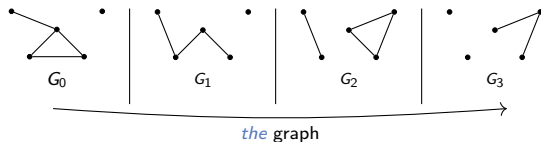
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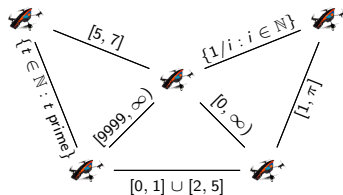
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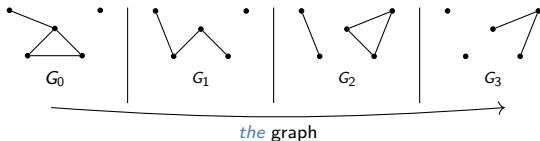
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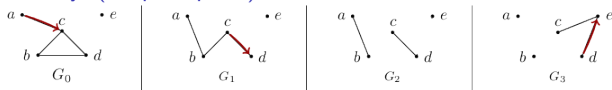
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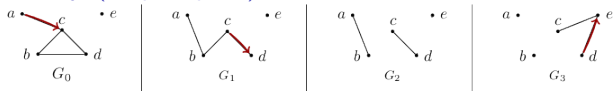
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## Journeys (temporal paths)



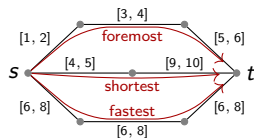
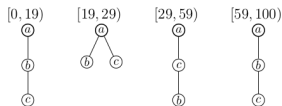
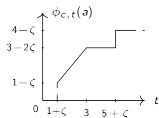
→ Temporal connectivity

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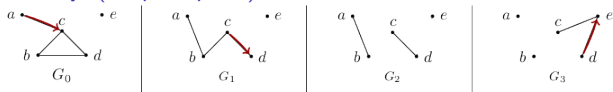


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## Temporal distance & shortest paths

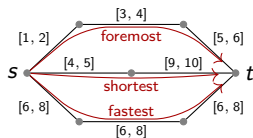
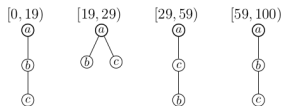
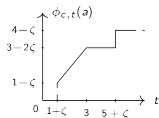


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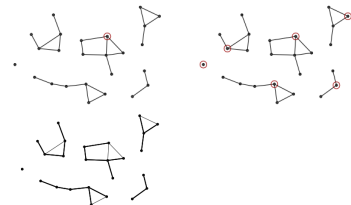


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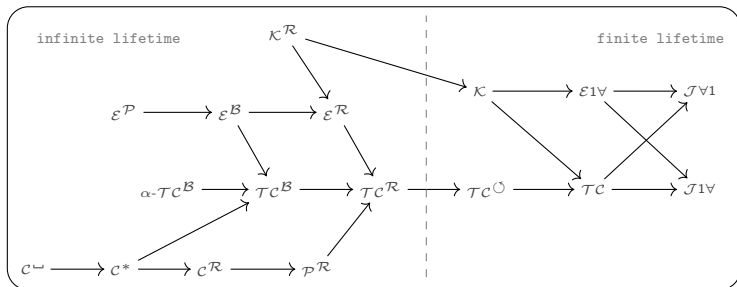
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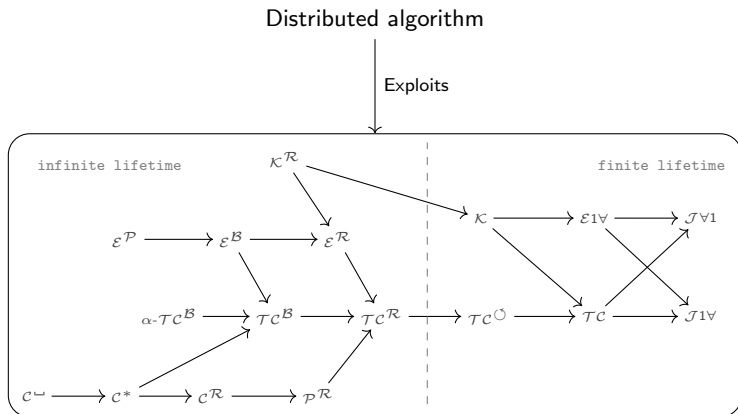
## Redefinition of classical problems



# Classes of dynamic networks/graphs

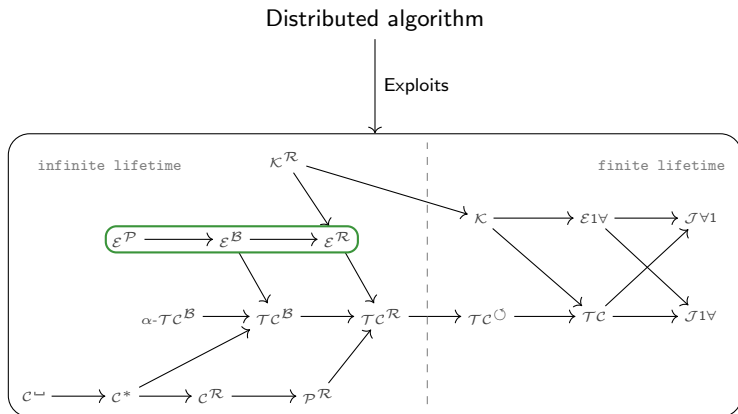


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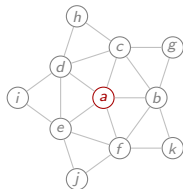
# Zoom: Optimal broadcast with termination detection (CHAP 2 SEC 2)

## Distributed problem:

→ A node must send a piece of information to all other nodes, then detect **termination**

→ Three criteria: **foremost**, **shortest**, **fastest**

Unsolvable without further assumption.



## Structure (+ knowledge)

- Connected footprint (obvious)
- Recurrent edges  $\mathcal{E}^{\mathcal{R}}$  (+  $n$ )
- Bounded-recurrent edges  $\mathcal{E}^{\mathcal{B}}$  (+  $\Delta$ )
- Periodic edges  $\mathcal{E}^{\mathcal{P}}$  (+  $p$ )

Note that  $\mathcal{E}^{\mathcal{P}} \subset \mathcal{E}^{\mathcal{R}} \subset \mathcal{E}^{\mathcal{B}}$

Casteigts *et al.*, Int. J. of Foundations of Computer Science, Vol. 26, Issue 4, 2015

(optimality metrics defined by Bui-Xuan, Ferreira, Jarry, 2003)

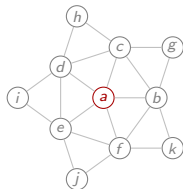
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## Theorems:

- Foremost feasible in  $\mathcal{E}^{\mathcal{R}}$
- Shortest feasible in  $\mathcal{E}^{\mathcal{B}}$  (but not in  $\mathcal{E}^{\mathcal{R}}$ )
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## Theorem:

$power(\mathcal{E}^{\mathcal{R}} + n) \subsetneq power(\mathcal{E}^{\mathcal{B}} + \Delta) \subsetneq power(\mathcal{E}^{\mathcal{P}} + p)$

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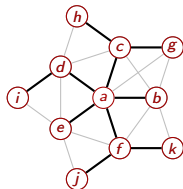
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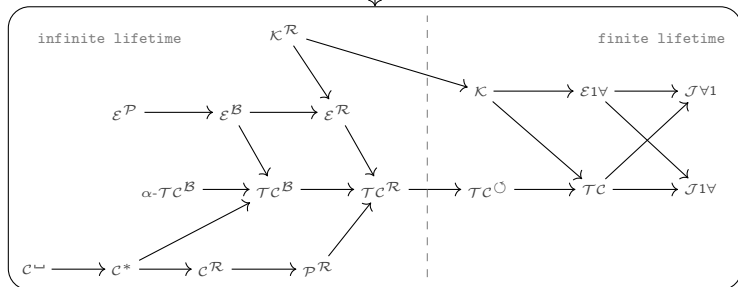
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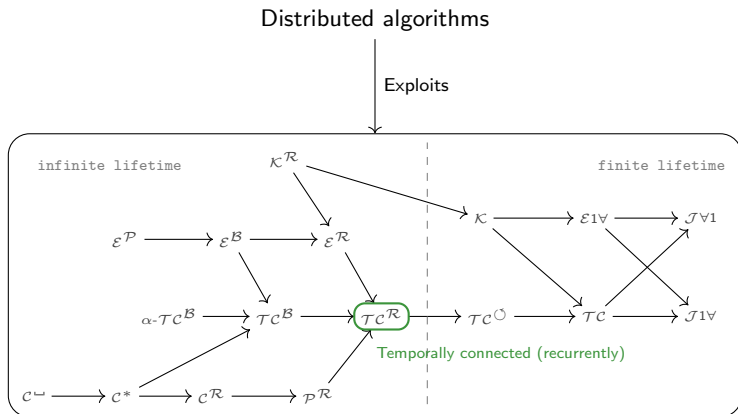
# Classes of dynamic networks

## Distributed algorithms

Exploits



# Classes of dynamic networks



$\mathcal{TC}^{\mathcal{R}} :=$  All nodes can reach each other through journeys infinitely often

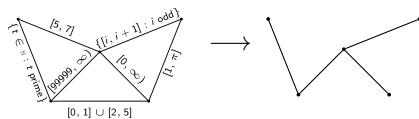
$$(\mathcal{TC}^{\mathcal{R}} := \forall t, \mathcal{G}_{[t, +\infty)} \in \mathcal{TC})$$



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Alternative characterization: **Eventual footprint connected**

Braud Santoni et al., 2016

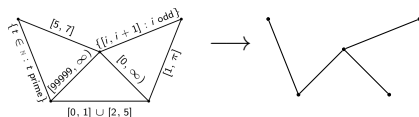


→ Can be exploited in a distributed algorithm Kaaouachi et al., 2016

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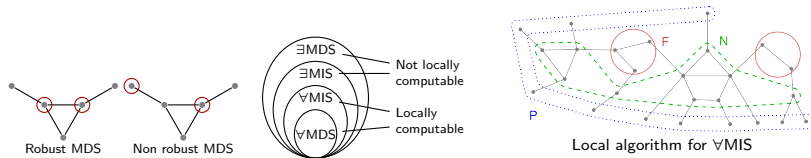
→ Can be exploited in a distributed algorithm Kaaouachi et al., 2016

## Robustness

(dealing with uncertainty)

→ New form of heredity in graphs: property/solution holds in all *connected spanning subgraph*

Ex: MINIMALDOMINATINGSET (MDS) and MAXIMALINDEPENDENTSET (MIS)

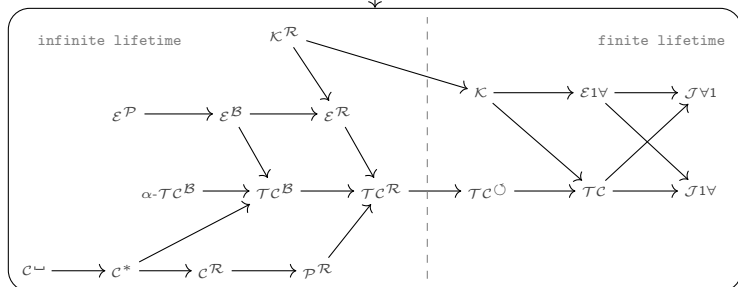


Casteigts, Dubois, Petit, Robson, CoRR, abs/1703.03190v2, 2018

# Classes of dynamic networks

## Distributed algorithms

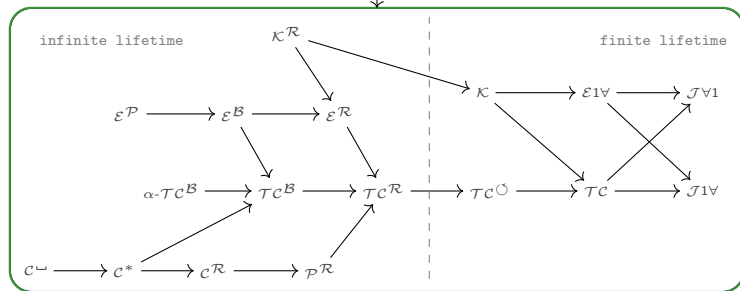
Exploits



# Classes of dynamic networks

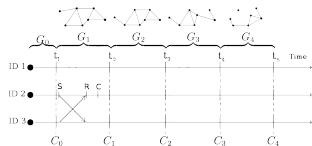
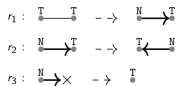
## Distributed algorithms

Exploits



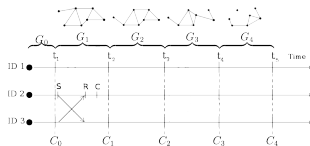
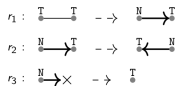
What if no structure at all?

## Maintaining a Spanning Forest



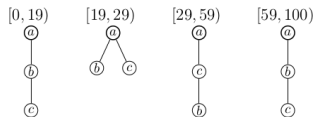
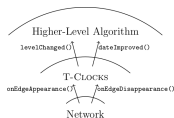
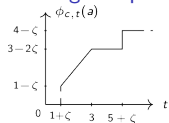
The Computer Journal (Oxford Univ. Press), to appear, 2018

## Maintaining a Spanning Forest



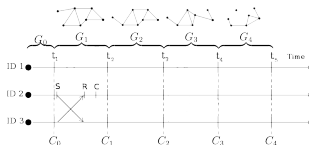
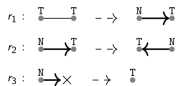
The Computer Journal (Oxford Univ. Press), to appear, 2018

## Measuring Temporal Distances



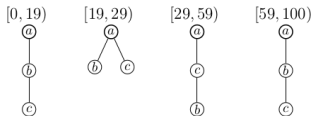
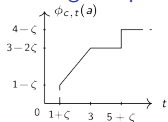
IEEE Transactions on Computers, Vol. 63, Issue 2, 2014

## Maintaining a Spanning Forest



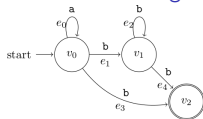
The Computer Journal (Oxford Univ. Press), to appear, 2018

## Measuring Temporal Distances



IEEE Transactions on Computers, Vol. 63, Issue 2, 2014

## The Power of Waiting



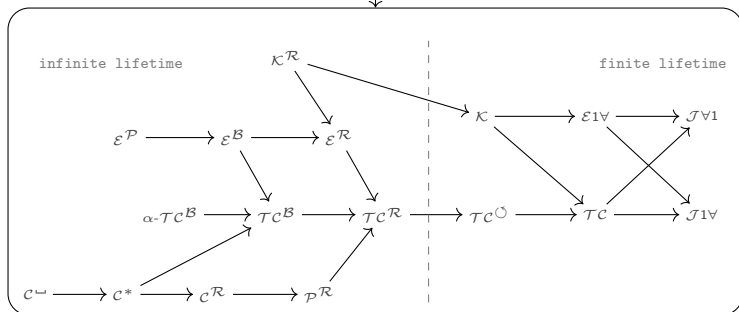
$e$	Presence $\rho(e, t) = 1$ iff	Latency $\zeta(e, t) =$
$e_0$	always true	$(p-1)t$
$e_1$	$t > p$	$(q-1)t$
$e_2$	$t \neq p^i q^{i-1}, i > 1$	$(q-1)t$
$e_3$	$t = p$	any
$e_4$	$t = p^i q^{i-1}, i > 1$	any

Theoretical Computer Science (Elsevier), Vol. 590, 27-37, 2015

# Classes of dynamic networks

## Distributed algorithms

Exploits

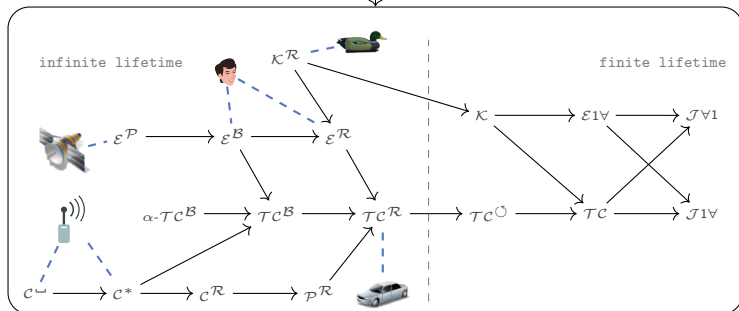




# Classes of dynamic networks

## Distributed algorithms

Exploits

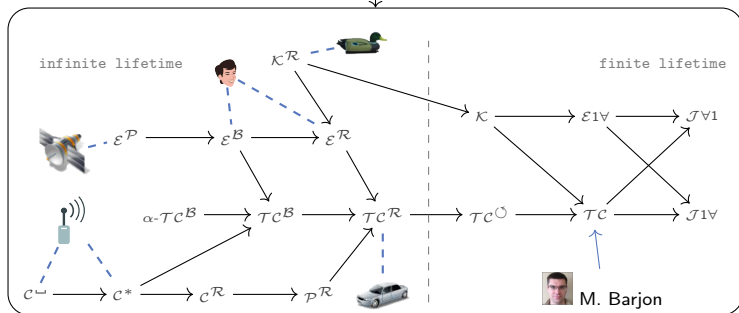


What about real-world mobility?

# Classes of dynamic networks

## Distributed algorithms

Exploits



Tests



Y. Neggaz

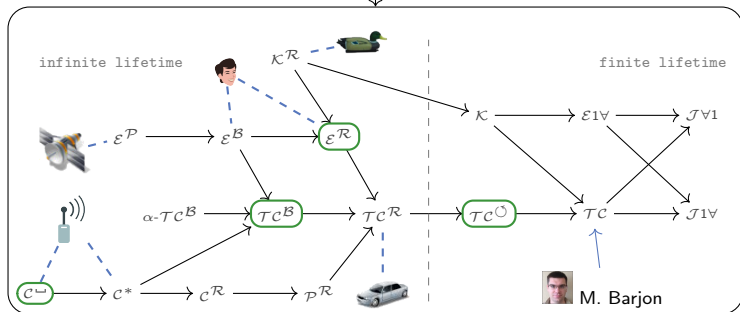
Offline analysis

What about real-world mobility?

# Classes of dynamic networks

## Distributed algorithms

Exploits



Tests



Y. Neggaz

Offline analysis

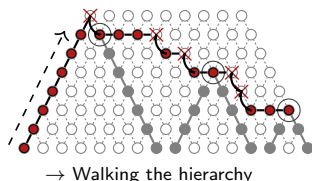
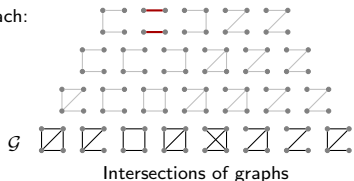
What about real-world mobility?

Ex: T-interval connectivity ( $\mathcal{C}^-$ )

All consecutive  $T$  graphs contain a common spanning tree  $\rightarrow$  measures stability

The problem, finding  $T$  (given a sequence of  $\delta$  graphs)

Approach:



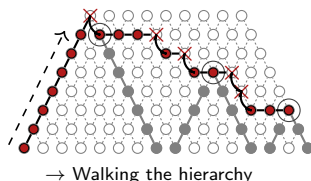
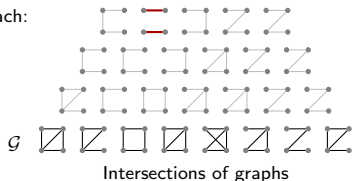
$\rightarrow$  **Theorem:**  $O(\delta)$  high-level operations (vs.  $O(\delta^2)$  naive)  
(intersections and connectivity tests).

Ex: T-interval connectivity ( $\mathcal{C}^{\sqcup}$ )

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Approach:



$\rightarrow$  **Theorem:**  $O(\delta)$  high-level operations (vs.  $O(\delta^2)$  naive)  
(intersections and connectivity tests).

Genericity:

	Property	composition	test	goal
$\mathcal{C}^{\sqcup}$	T-interval connectivity	intersection	connectivity	max
$\mathcal{E}^{\mathcal{R}}$	Realization of the footprint	union	identity	min
$\mathcal{TC}^{\mathcal{B}}$	Temporal diameter	concat TC	completeness	min
$\mathcal{TC}^{\circ}$	Round-trip temp. diameter	concat RTTC	completeness	min

Casteigts *et al.*, 9th Int. Conference on Algorithms and Complexity (CIAC), 2015

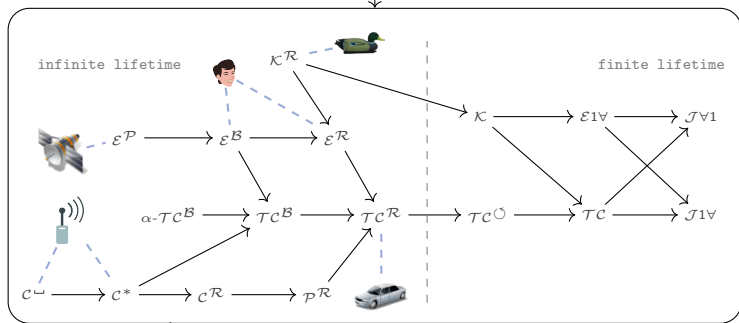
Casteigts *et al.*, 19th Int. Conference on Structural Information and Communication Complexity (SIROCCO), 2017

Combined article in minor revision (in ToCS)

# Classes of dynamic networks

Distributed algorithms

Exploits



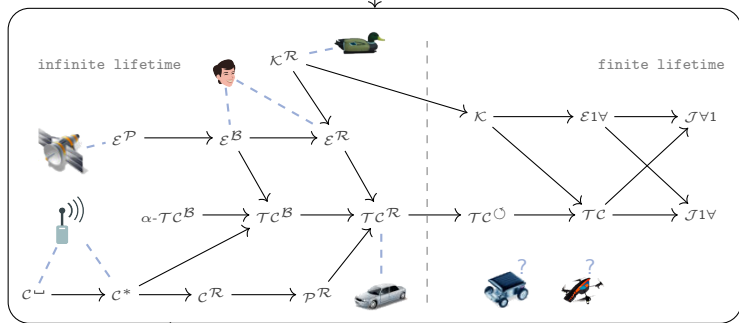
Tests

Offline analysis

# Classes of dynamic networks

Distributed algorithms

Exploits



Tests

What if we can control movements?

Offline analysis



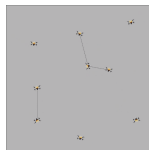


# Collective movements which induce temporal structure

Jason Schoeters (since Nov. 2017)  
PhD funded by ANR ESTATE



Synthesizing collective movements (a.k.a. *mobility models*) so as to satisfies temporal properties on the resulting communication graph.



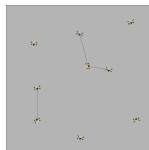
← this network  $\in \mathcal{E}^{\mathcal{R}}$

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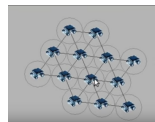


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← this network  $\in \mathcal{E}^{\mathcal{R}}$

this one  $\in \mathcal{C}^*$  →

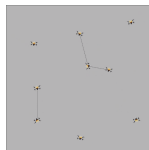


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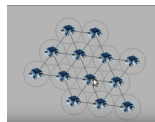


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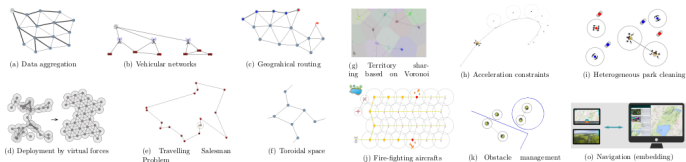
Main objectives:

- ▶ Design mobility models independently (warm up)
- ▶ Combine them with concrete problems like exploration (more difficult)

Interesting target:  $\mathcal{TC}^{\mathcal{B}}$  (bounded temporal diameter)

→ Weakest setting to detect a crash.

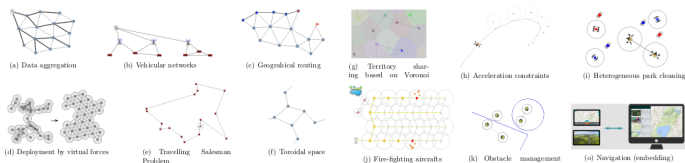
# JBOTSIM



Interactive, simple to use, extensible, event-driven programming (java)

→ Download statistics (SF): 150 (2015), 900 (2016), 1100 (2017), ...

Growing community, ~ 10 "universities"



Interactive, simple to use, extensible, event-driven programming (java)

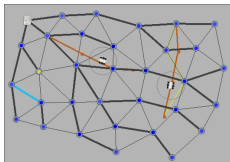
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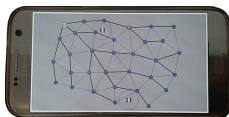
Enables the use of many models of computations (by design), at graph or network level.

Masters course “Algorithmique de la mobilité” (48h, Bordeaux) (others in Ottawa, Marseille, Strasbourg)

Ex: Student project (2017)

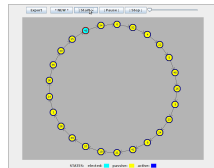


JBotsim on Android!

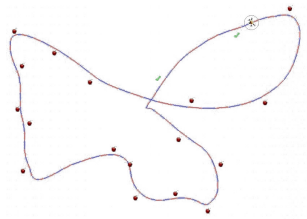


Kinda Al Chahid (M2)

DAVIS project (E.Godard)



## Perspectives



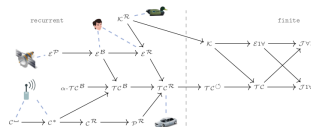
# Perspectives

Two natural perspectives:

## Structure in dynamic networks (a step further)

- Explore relations among existing and new classes of dynamic networks
- Focus on  $\mathcal{TC}^{\mathcal{R}}$  and  $\mathcal{TC}^{\mathcal{B}}$  and **robustness**
- Consider studying real-world **data sets**

(Perspective 0)



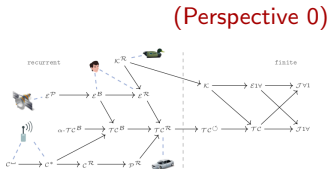


# Perspectives

Two natural perspectives:

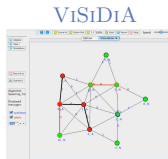
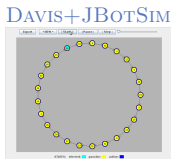
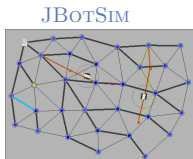
## Structure in dynamic networks (a step further)

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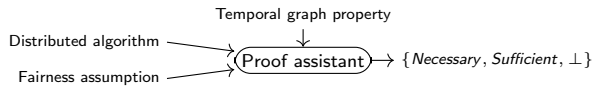
## Around JBotSim

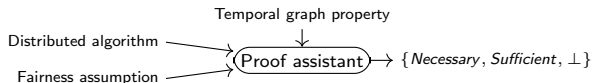
→ **Convergence** of tools



- Specific extensions, e.g. **Remote topology viewer** through a classroom network
- Interactive web platform based on **Jupyter**?

A one-year postdoc is coming (starting Sep. 1, 2018)





## Current related efforts:

- with Event-B (F. Fakhfakh, D. Mery, M. Mosbah, M. Tounsi)  
Ex: Proving correctness of our spanning forest algorithm by refinements
- PADEC: Coq library (K. Altisen, P. Corbineau, S. Devismes) – ANR ESTATE  
Ex: Proving correctness of algorithms in the “Locally shared memory” model

## Past efforts:

- LOCO: Coq library (P. Castéran) Ex: Local computation à la Métivier

## Ambition:

Prove formally that a given property on the dynamics is necessary or sufficient to a given algorithm.

- Need to pair up with a partner!

## 1) Collective movements which induce temporal structure

Already discussed

Synthesize collective movements (*a.k.a.* *mobility models*) such that the resulting graph satisfies temporal properties.

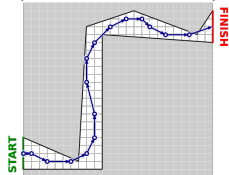
## 1) Collective movements which induce temporal structure

Already discussed

Synthesize collective movements (*a.k.a.* *mobility models*) such that the resulting graph satisfies temporal properties.

## 2) Integrating physical constraints in a tractable way

Discrete acceleration models  
VECTOR RACER  
(paper and pencil game)



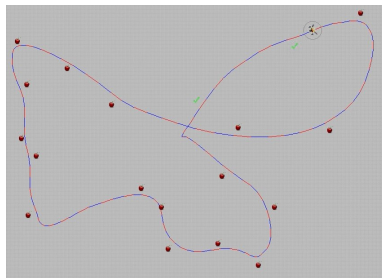
→ Impact on problems, e.g. TSP

### Ambition:

→ Occupy the space between **control theory** and **empirical approaches**

**Discrete algorithms!**

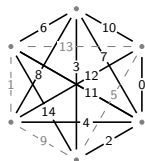
Exploratory work with J. Schoeters and M. Raffinot



**Theorem:** Acceleration does impact the visit order!

## Simplification of temporal cliques

Setting: Complete graph, every edge exists only one instant, remove as many edges as possible while remaining temporally connected (i.e., in  $\mathcal{TC}$ ).



**Theorem:**  $O(n)$  edges can be removed

Akrida *et al.* 2015

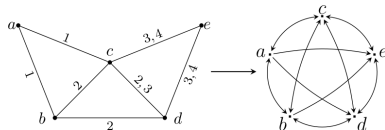
**Theorem:** All but  $O(n \log n)$  edges can be removed!

Casteigts *et al.* 2018  
(in preparation)

Open question: Is it optimal? Could we remove all but  $O(n)$ ?

## Transitive closures of journeys

Def: journey in  $\mathcal{G} \iff$  arc in transitive closure.



Question: What is the set of possible transitive closures?

# Acknowledgments

## PhD students (supervised or co-supervised):

- ▶ Jason Schoeters
- ▶ Matthieu Barjon and Yessin M. Neggaz (with C. Johnen and S. Chaumette)



## PhD students (through collaboration):

- ▶ Carlos Gómez Calzado (3 month visit in Bordeaux)
- ▶ Jérémie Albert, Ahmed Jedda, Walter Quattrociochi (joint work in Ottawa)

## Masters students:

- ▶ Kinda Al Chahid (current)
- ▶ Robin Despouys, David Del Campo + 4 of the above

## Projects (significant scale):

- ▶ DRDC W7714-115111/001/SV (Defence Research and Development Canada) 
- ▶ ANR ESTATE (Enhancing Safety and Self-Stabilization in Time-Varying Distributed Environments) 

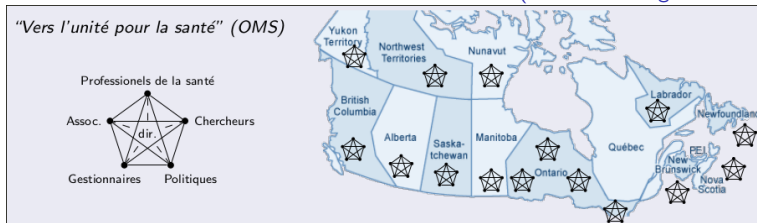
## Other co-authors (alphabetical order):

Frédéric Amblard, Lionel Barrère, Jean-Marie Berthelot, Louise Bouchard, Mariette Chartier, Marie-Hélène Chomienne, Swan Dubois, Afonso Ferreira, Paola Flocchini, Colette Johnen, Guy-Vincent Jourdan, Emmanuel Godard, Nishith Goel, Frédéric Guinand, Ralf Klasing, Alberto Lafuente, Mikel Larrea, Bernard Mans, Luke Mathieson, Hussein Mouftah, Yves Métivier, Amiya Nayak, Joseph Peters, Franck Petit, Yoann Pigné, Mike Robson, Nicola Santoro, Ivan Stojmenovic, Alain Trugeon, Jan Warnke, Mark Yamashita, and Akka Zemhari.



## Health Networks in Canada

(with a sociologist L. Bouchard)



## Differential Privacy for Linguistic Data

(with a physician M.H. Chomienne)

