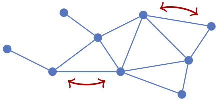
Distributed Computing in Dynamic Networks
–
Impact of the dynamics on definitions and feasibility

Arnaud Casteigts
–
University of Bordeaux

Distributed Computing



Collaboration of distinct entities to perform a common task.

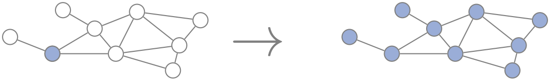
No centralization available. Direct interaction.

(Think globally, act locally)

Examples of problems

Broadcast

Propagating a piece of information from one node to all others.



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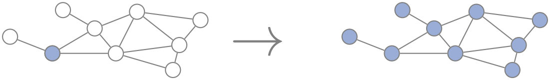
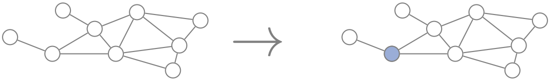


Election

Distinguishing exactly one node among all.



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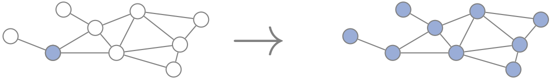

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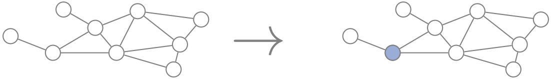
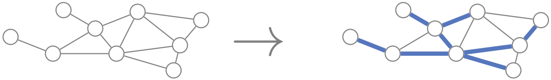


Spanning tree

Selecting a cycle-free set of edges that interconnects all nodes.



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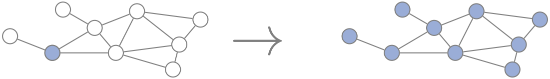

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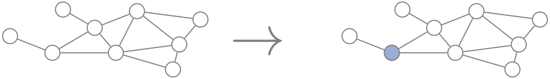

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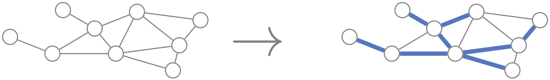
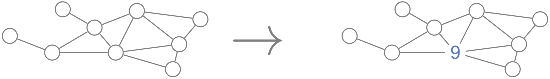


Counting

Determining how many participants there are.



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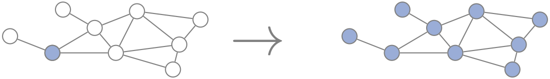

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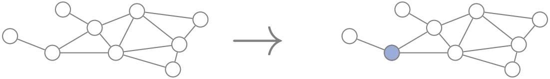

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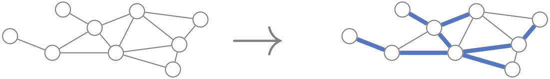

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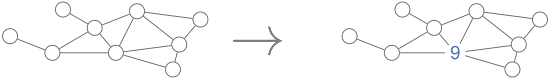
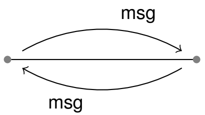


Consensus, naming, routing, exploration, ...

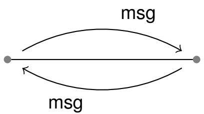
Communication Models

Communication Models



(a) Message passing

Communication Models



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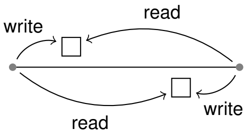
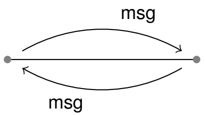


(b) Registers

Communication Models



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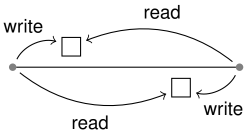

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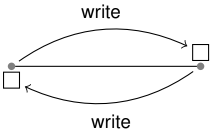
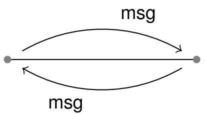


(c) Mailboxes

Communication Models



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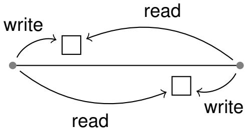

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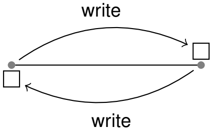

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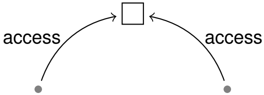
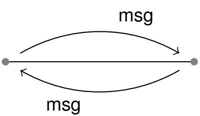


(d) Shared memory

Communication Models



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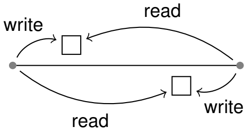

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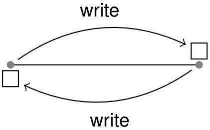

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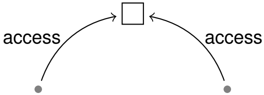

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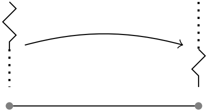

(e) Mobile agents

Abstracting Communications

Atomic interaction

(Population protocols *(Angluin et al., 2004)* ; Graph relabeling systems *(Litovsky et al., 1999)*)

Ex : $T \text{---} N \longrightarrow T \text{---} T$

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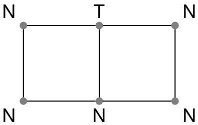

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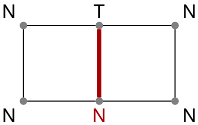

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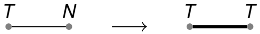
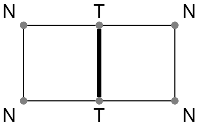
Ex : 



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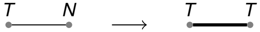
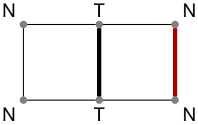
Ex : 



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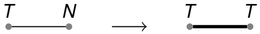
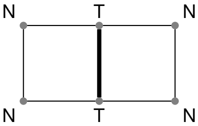
Ex : 



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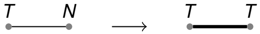
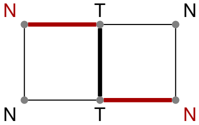
Ex : 



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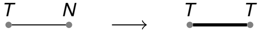
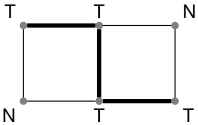
Ex : 



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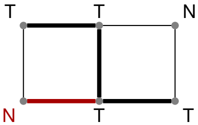

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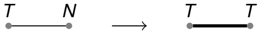
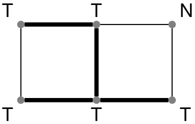
Ex : 



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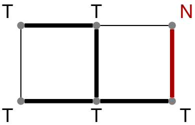

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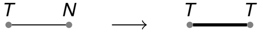
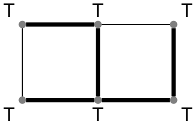
Ex : 



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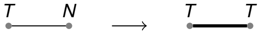
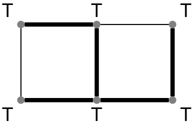
Ex : 



Note : Scheduling is not part of the algorithm !

$\longrightarrow$ Can be adversarial, randomized, etc.

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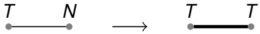

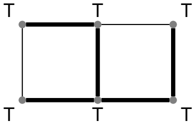

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Scope of the models

Relations between them *(Chalopin, 2006)*

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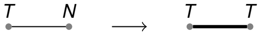

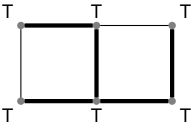

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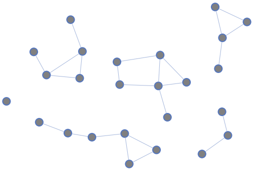
Dynamic Networks



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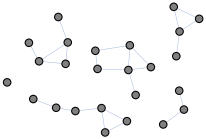

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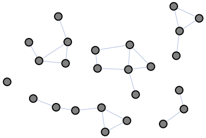
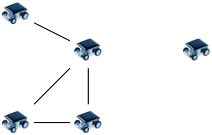


Example of scenario
(say, exploration by mobile robots)



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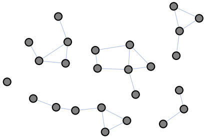

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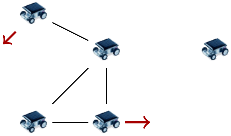

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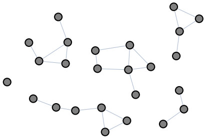

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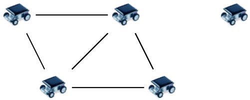

In fact, *highly* dynamic networks.

Ex : 

How changes are perceived ?

- ~~Faults and Failures ?~~
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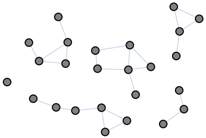

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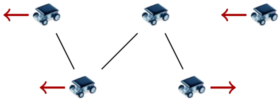

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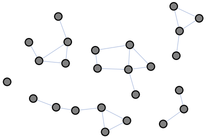

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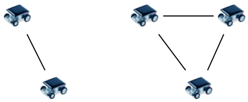

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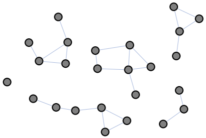

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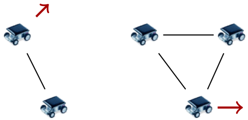

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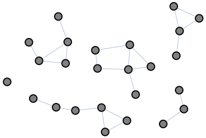

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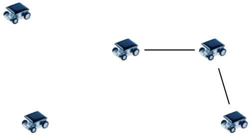

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Also called *evolving graphs* or *time-varying graphs*.

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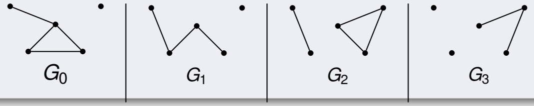

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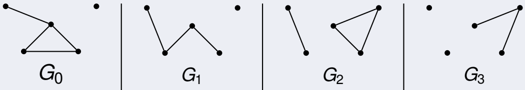

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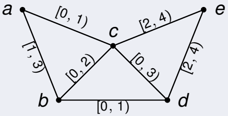
$\rho : E \times \mathcal{T} \to \{0, 1\}$



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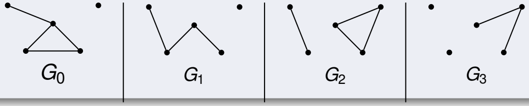

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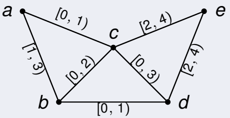

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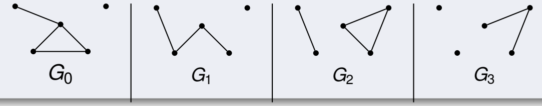

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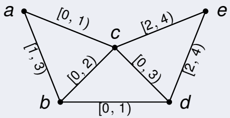

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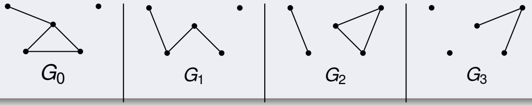

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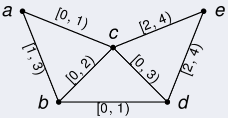

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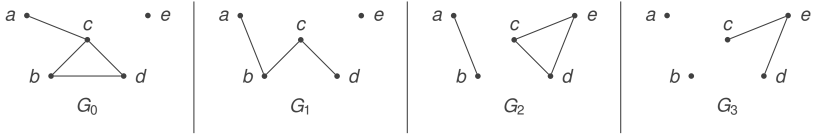
For references, see *(Ferreira, 2004)* and *(C., Flocchini, Quattrociocchi, Santoro, 2012)*

Basic graph concepts



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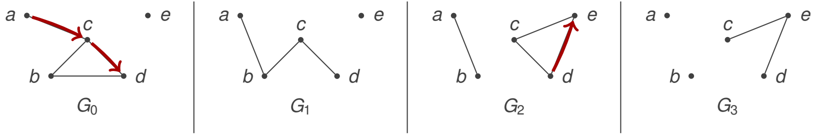

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Ex : $((ac, t_1), (cd, t_2), (de, t_3))$ with $t_{i+1} \geq t_i$ and $\rho(e_i, t_i) = 1$

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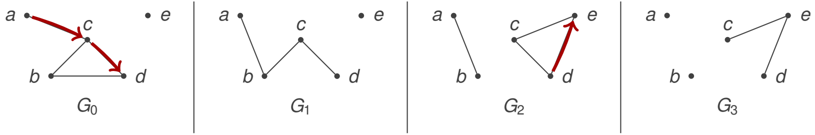

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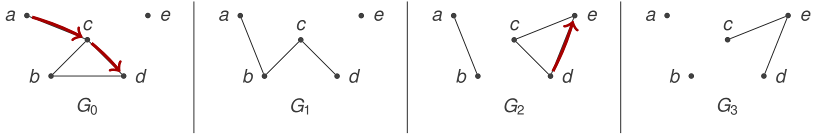

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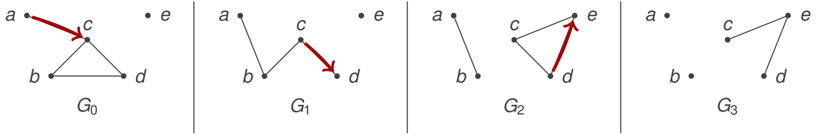

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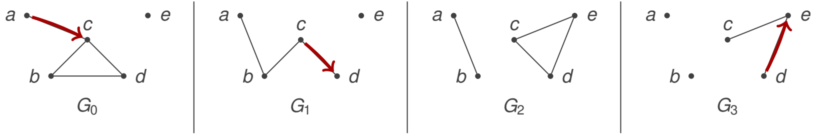

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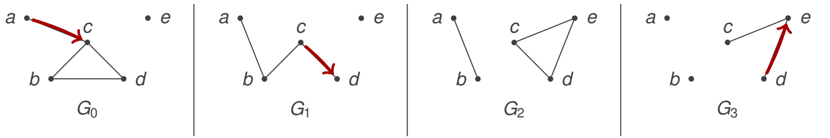

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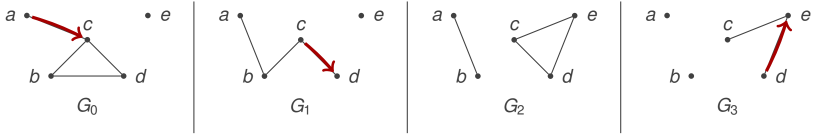

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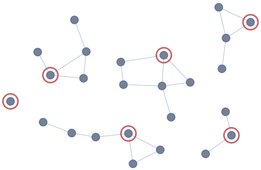
Many other concepts... (ask me !).

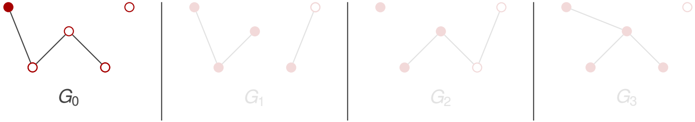
Topological assumptions for distributed algorithms



Feasibility, Necessary and sufficient conditions, ...

Informal example

Ex : Broadcast algorithm 



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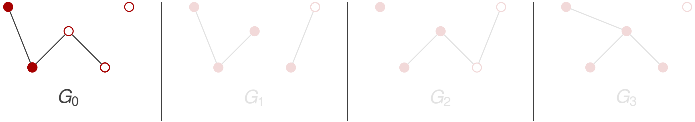

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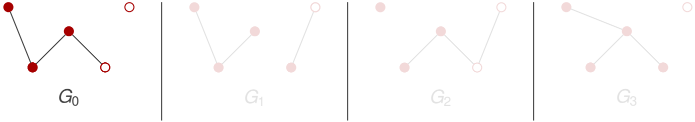

Lucky version.

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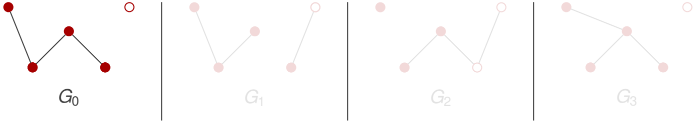

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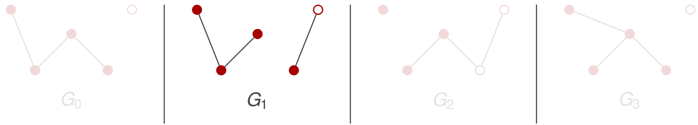

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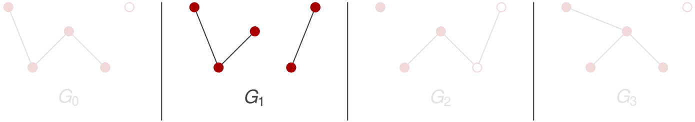

Lucky version. Yeah ! !

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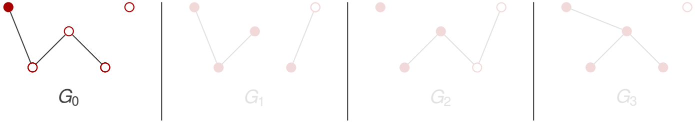

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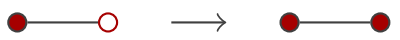

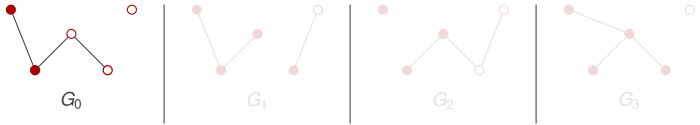

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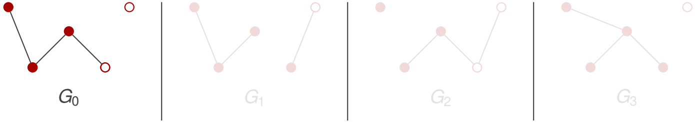

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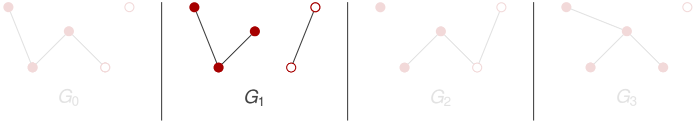

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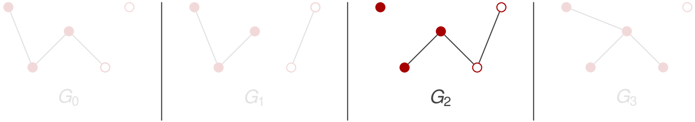

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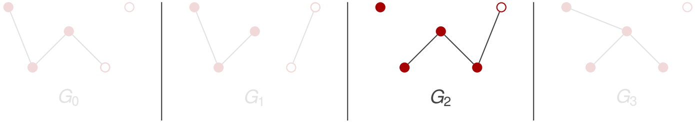

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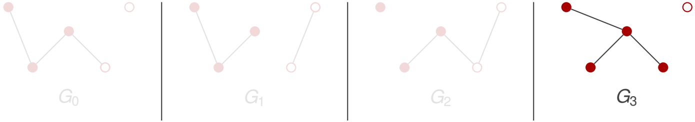

But things could have gone differently. Too late ! Failure !

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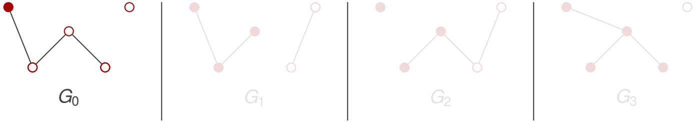

Or even worse..

Informal example

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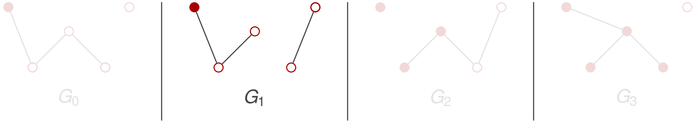

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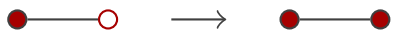
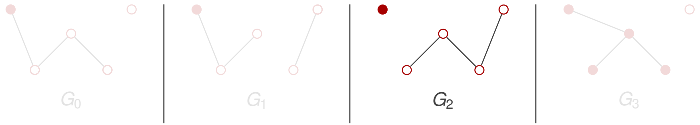
Ex : Broadcast algorithm 



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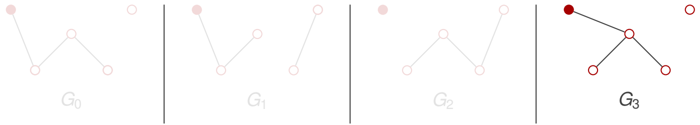

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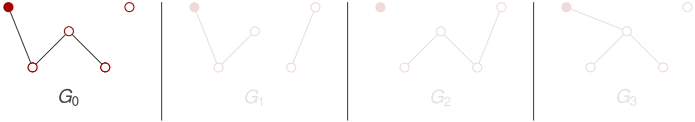

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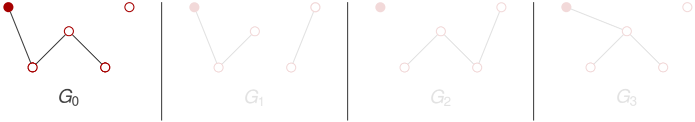

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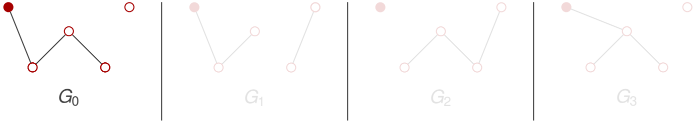

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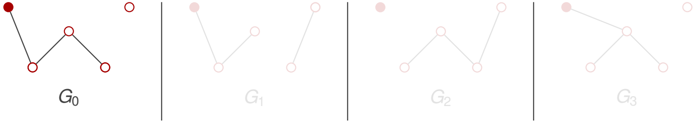

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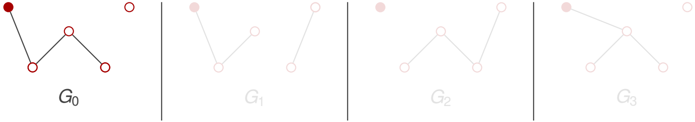

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$\rightarrow$ Is the success <u>possible</u> ? Of course, but why ?

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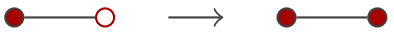
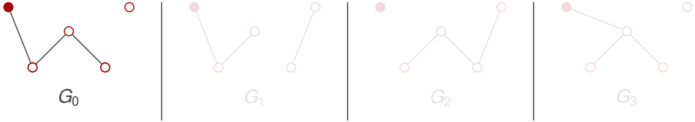
Ex : Broadcast algorithm 



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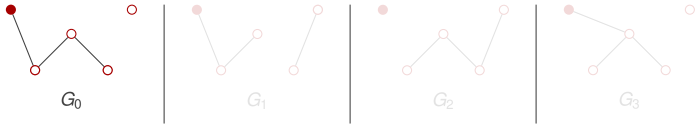

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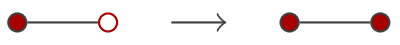
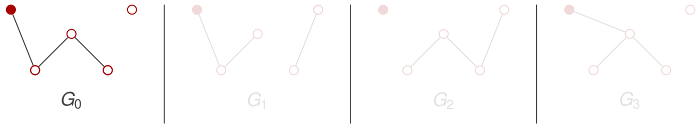
Ex : Broadcast algorithm 



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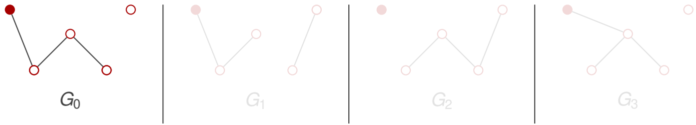

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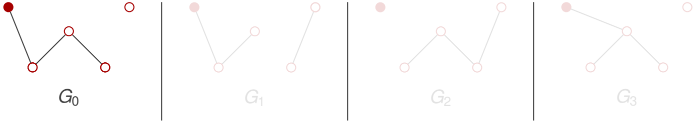

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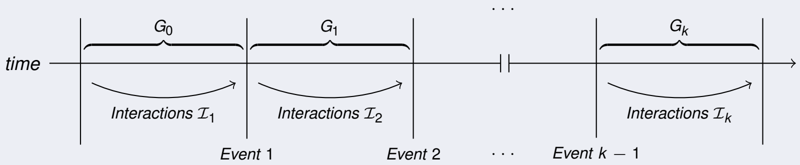
More formally...

Interaction over dynamic graphs

Interactions over a Dynamic Graph $\mathcal{G} = \{G_0, G_1, ..., G_k\}$



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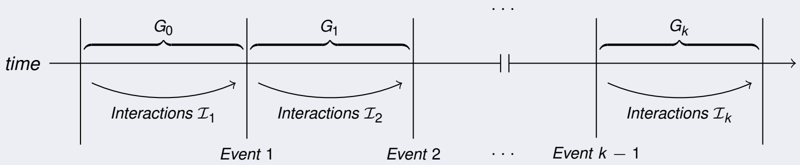

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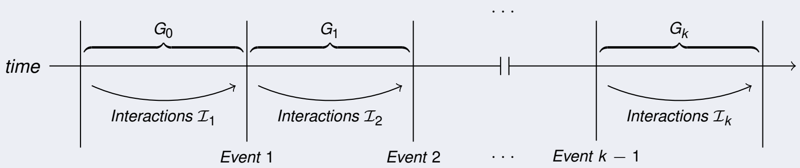

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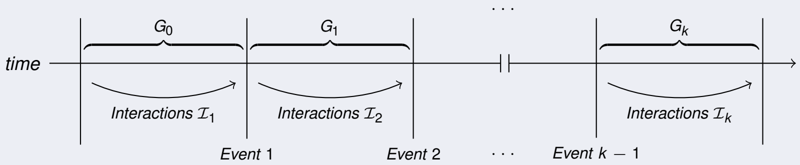

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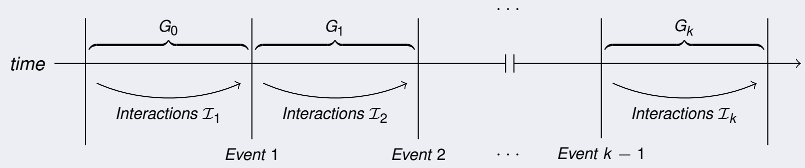

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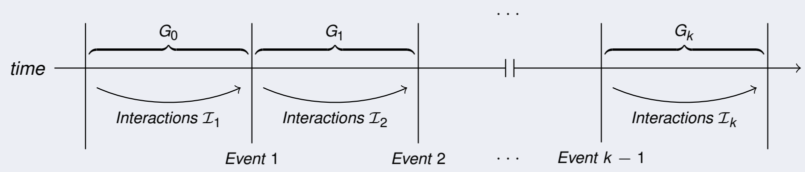

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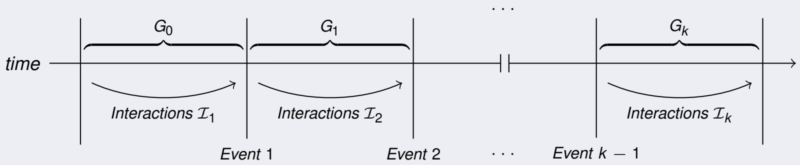

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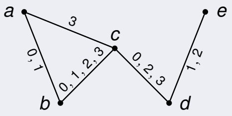

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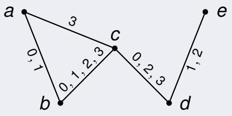

$\in \mathcal{C}_2 \cap \mathcal{C}_3$

Counting algorithm (non-uniform)

Counting with a distinguished counter

- Initial states : 1 for the counter, N for all other nodes.
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$\rightarrow$ $\mathcal{C}_6$: all the nodes verify $\mathcal{P}$, (noted $*$–$*$).

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Conditions and classes of graphs

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  - $\rightarrow$ $\mathcal{C}_7$: graphs having this property.
- Sufficient condition $\mathcal{C}_{\mathcal{S}}$: all pairs of nodes must share an edge at least once over time ($*{-}*$). *(Marchand de Kerchove, Guinand, 2012)*
  - $\rightarrow$ $\mathcal{C}_6$ (already seen before).

Tightness of a condition ? *(Marchand de Kerchove, Guinand, 2012)*

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- Not satisfied $\implies$ failure is guaranteed $(\nexists X, success(X))$

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Ex. uniform counting (last algorithm) :

$\rightarrow$ $(*\rightsquigarrow 1)$ is a tight necessary condition

$\rightarrow$ $(*{-}*)$ is a tight sufficient condition

In between : outcome is uncertain... might succeed or fail.

Classifying dynamic networks

$1 - *$ $\mathcal{C}_5$

$1 \overset{st}{\rightsquigarrow} *$ $\mathcal{C}_3$

$1 \rightsquigarrow *$ $\mathcal{C}_1$

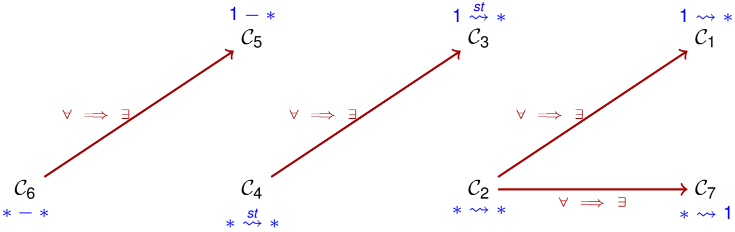
$\mathcal{C}_6$ $* - *$

$\mathcal{C}_4$ $* \overset{st}{\rightsquigarrow} *$

$\mathcal{C}_2$ $* \rightsquigarrow *$

$\mathcal{C}_7$ $* \rightsquigarrow 1$

Classifying dynamic networks



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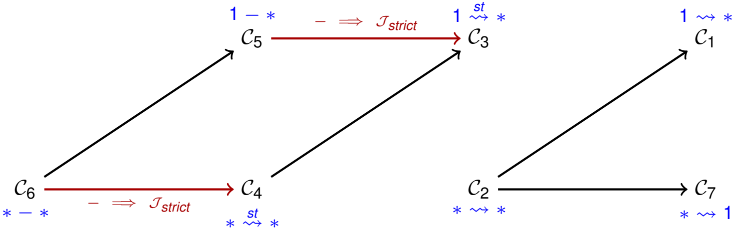

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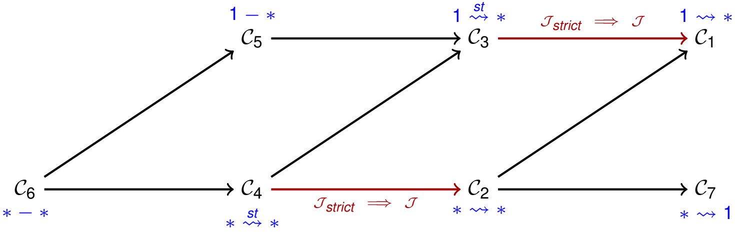

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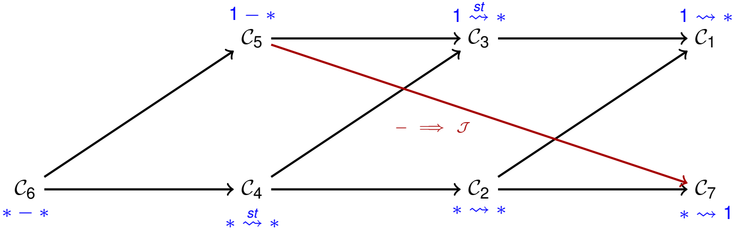

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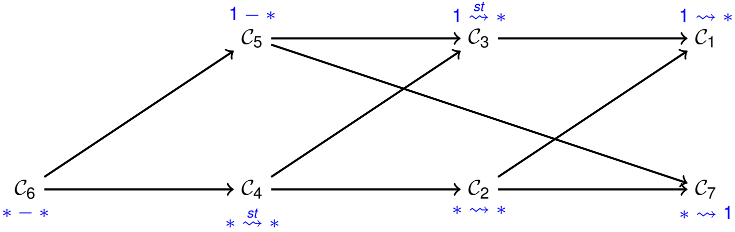
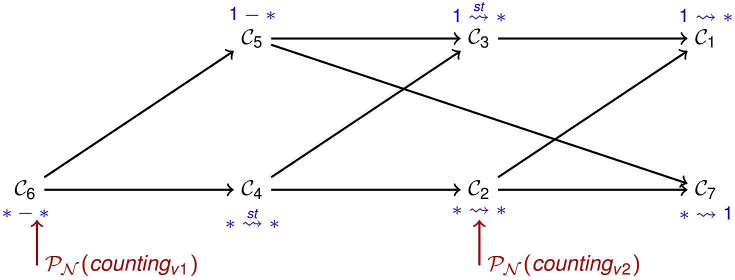


Classifying dynamic networks

$$
\begin{tikzcd}
 & \overset{1 - *}{\mathcal{C}_5} \arrow[r] \arrow[drr] & \overset{1 \overset{st}{\rightsquigarrow} *}{\mathcal{C}_3} \arrow[r] & \overset{1 \rightsquigarrow *}{\mathcal{C}_1} \\
\underset{* - *}{\mathcal{C}_6} \arrow[ur] \arrow[r] & \underset{* \overset{st}{\rightsquigarrow} *}{\mathcal{C}_4} \arrow[ur] \arrow[r] & \underset{* \rightsquigarrow *}{\mathcal{C}_2} \arrow[ur] \arrow[r] & \underset{* \rightsquigarrow 1}{\mathcal{C}_7}
\end{tikzcd}
$$

$\rightarrow$ Comparison of algorithms on a formal basis



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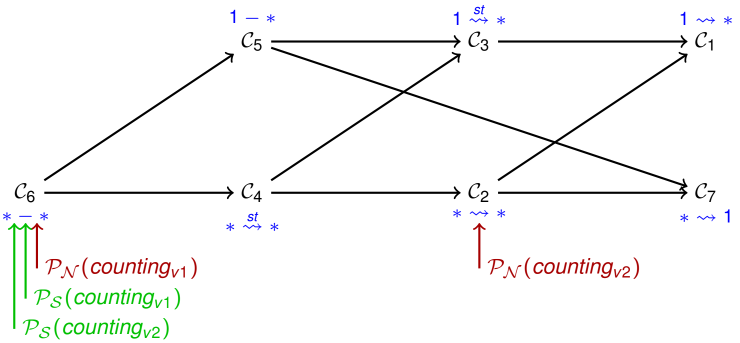
Classifying dynamic networks



$\longrightarrow$ Comparison of algorithms on a formal basis

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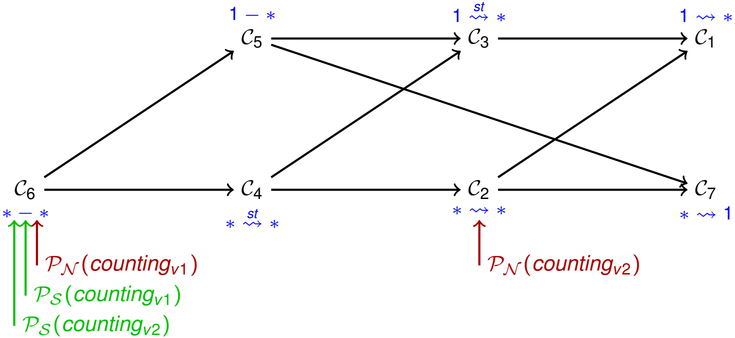

→ Comparison of algorithms on a formal basis

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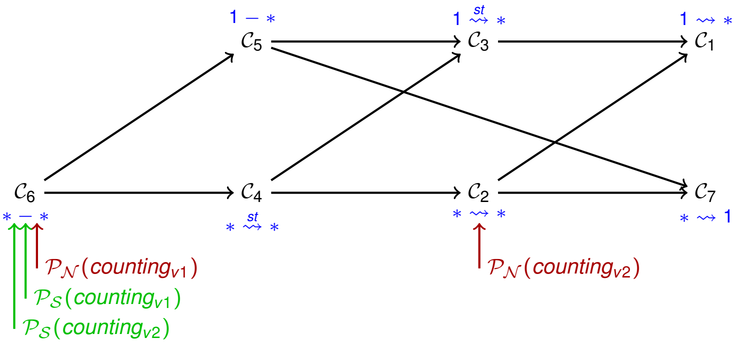
Classifying dynamic networks



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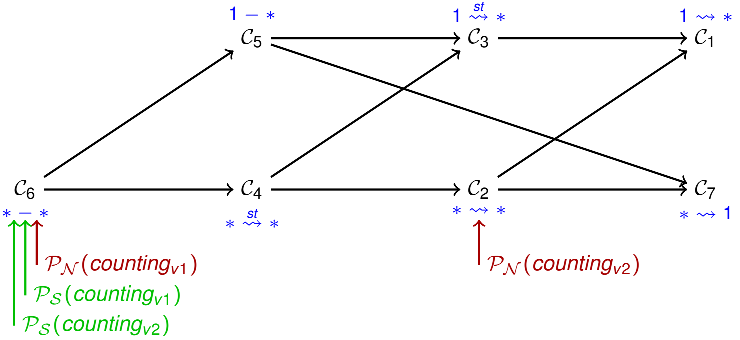

→ Comparison of algorithms on a formal basis

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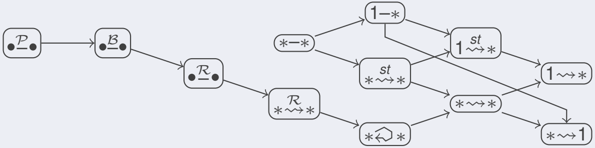
Q : How far beyond toy examples ?

(C., Flocchini, Quattrociocchi, Santoro, 2012)

Extending the hierarchy



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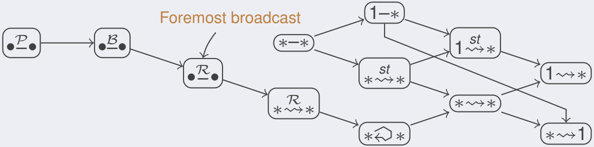
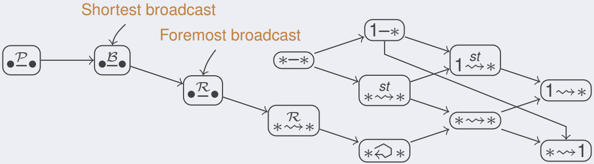


Classifying further

(C., Flocchini, Quattrociocchi, Santoro, 2012)

Extending the hierarchy



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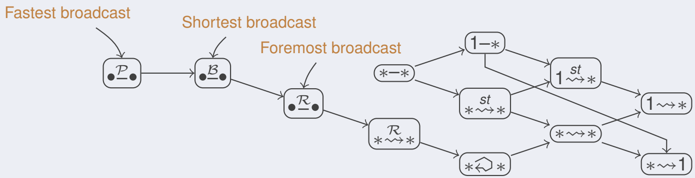

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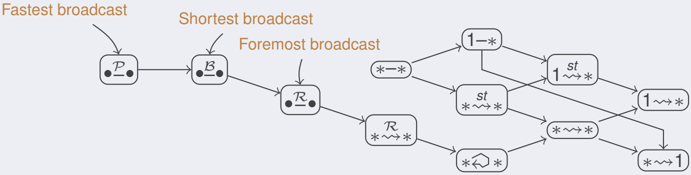

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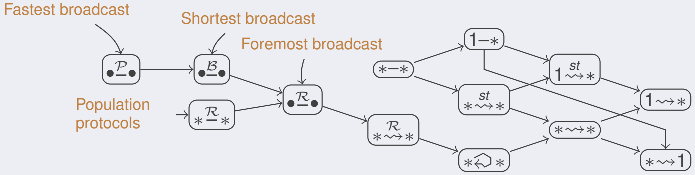

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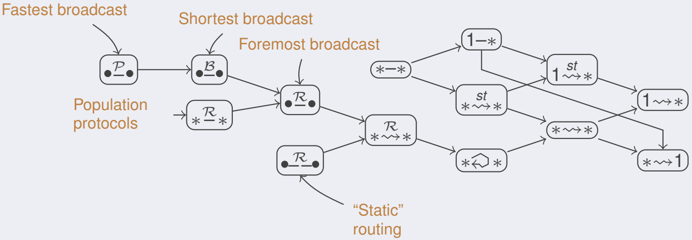

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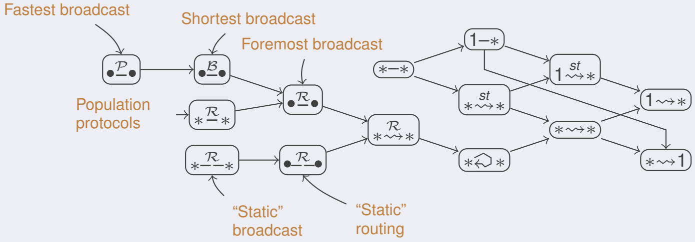

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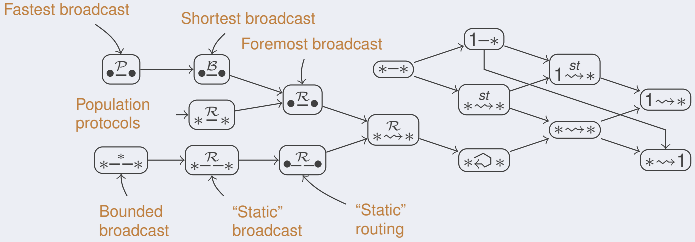
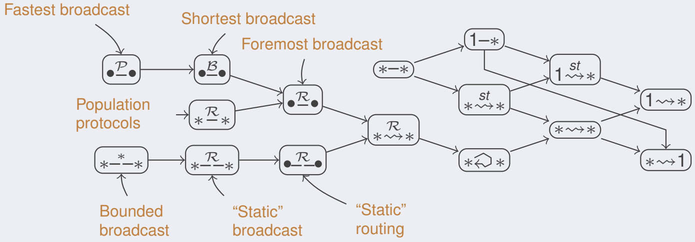


Classifying further

(C., Flocchini, Quattrociocchi, Santoro, 2012)

Extending the hierarchy



Ex : Bounded broadcast in $(*\overset{*}{-}*)$

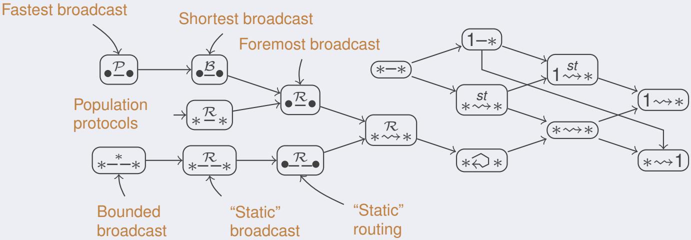
(O'Dell and Wattenhofer, 2005)

The graph is arbitrarily dynamic, as long as every G_i remains connected :

Classifying further

(C., Flocchini, Quattrociocchi, Santoro, 2012)

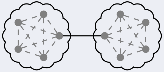
Extending the hierarchy



Ex : Bounded broadcast in ($*\overset{*}{-\!-}*$) *(O'Dell and Wattenhofer, 2005)*

The graph is arbitrarily dynamic, as long as every G_i remains connected :

Min cut of size 1 between informed and uninformed nodes :
$\longrightarrow$ At least one new node informed in each step.



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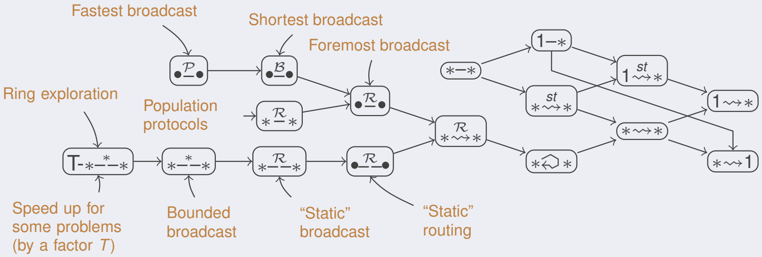

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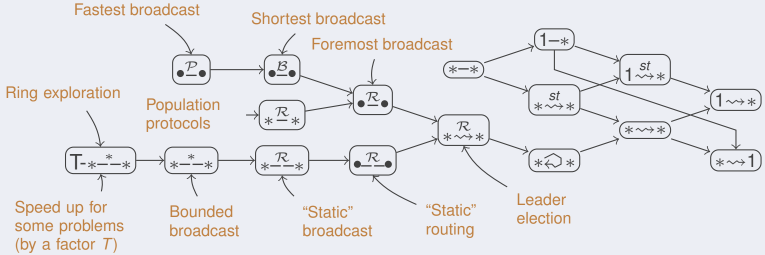

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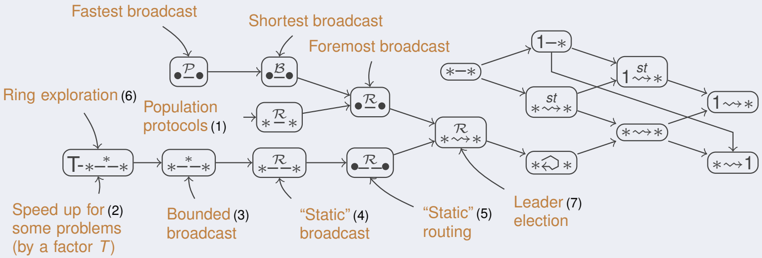


(1) Complete graph of interaction *(Angluin, Aspnes, Diamadi, Fischer, Peralta, 2004)*

(2) T-interval connectivity *(Kuhn, Lynch, Oshman, 2010)*

(3) Constant connectivity *(O'Dell and Wattenhofer, 2005)*

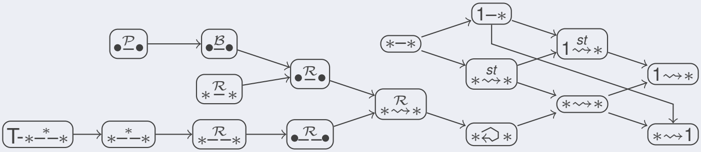
(4) Eventual instant connectivity *(Ramanathan, Basu, and Krishnan, 2007)*

(5) Eventual instant routability *(Ramanathan, Basu, and Krishnan, 2007)*

(6) T-interval connectivity *(Ilcinkas, Wade, 2013)*

(7) Recurrent temporal connectivity *(Arantes, Greve, Sens, Simon, 2013)* *(Gòmez-Cazaldo, Lafuente, Larrea, Raynal, 2013)*

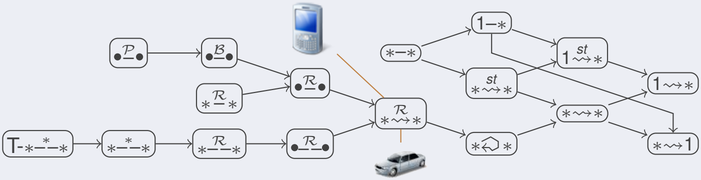
Real mobility contexts



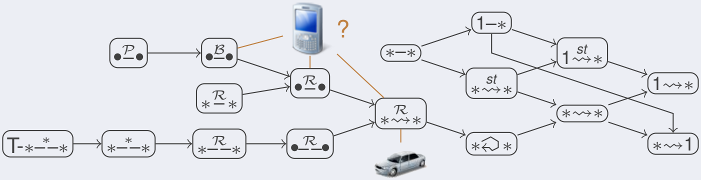
Classifying further

(C., Flocchini, Quattrociocchi, Santoro, 2012)

Real mobility contexts



Real mobility contexts



Classifying further

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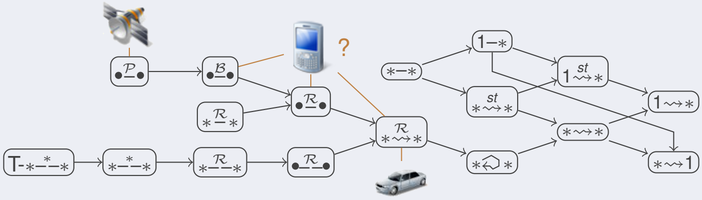
Real mobility contexts



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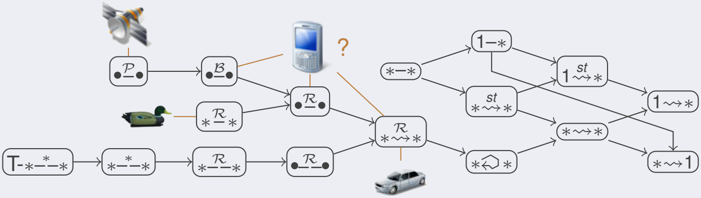
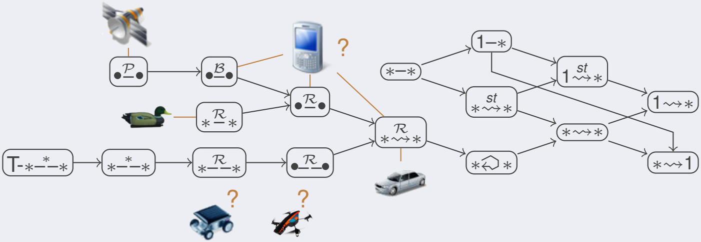
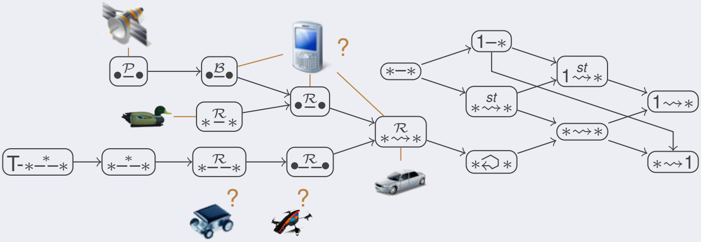


Real mobility contexts



Real mobility contexts



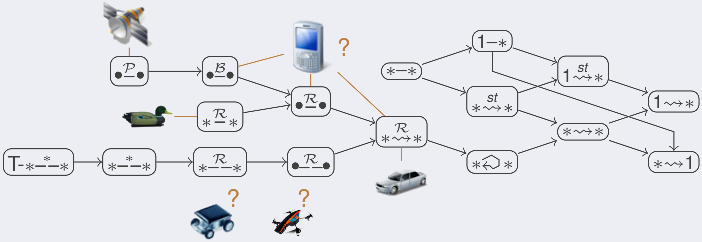
How to proceed ?

→ Generate connection traces

→ Test properties

Classifying further *(C., Flocchini, Quattrociocchi, Santoro, 2012)*

Real mobility contexts



How to proceed ?

→ Generate connection traces

→ Test properties

- Temporal Connectivity *(Whitbeck et al. 2012 ; Barjon et al., 2014)*
- T-Interval Connectivity *(C. et al., 2014)*

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